Surface Parameter Estimation by Inverse Modeling of Ground Penetrating Radar

Thesis submitted in partial fulfillment
Of the requirements for the degree of

MASTER OF TECHNOLOGY
In
Communication & Signal Processing
By

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Roll No: 211EC4001

Department of Electronics & Communication Engineering

NIT Rourkela, 2013

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Under the Guidance of
Prof. S. Maiti

Department of Electronics & Communication Engineering

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Dedicated to My Parents

National Institute of Technology

Rourkela

CERTIFICATE

This is to certify that the work in the thesis entitled **"Surface Parameters Estimation by Inverse Modeling of Ground Penetrating Radar" by Rupam Kumari** is a record of an original research work done by her during 2012-2013 under my supervision and guidance in partial fulfillment of the requirement for the award of the degree of **Master of Technology** with the specialization of Communication & Signal Processing in the Department of Electronics & Communication Engineering, National Institute of Rourkela. The result incorporated in the thesis has not been submitted for award of any degree elsewhere.

Place: Rourkela

Date:

Prof. S. Maiti

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Abstract

Ground Penetrating Radar is a high resolution electromagnetic technique to analyse the soil sub-surface. This requires accurate forward modeling of the GPR soil sub-surface system. Depending upon the application and the environment in which GPR is to be employed several forward analytical or numerical modeling methods are proposed by the researchers. Some of the analytical methods include standard reflection coefficient modeling, full waveform inversion modeling, transmission line modeling etc. Inverse modeling of GPR, which is getting back the electromagnetic parameters of targets based on the electromagnetic responses of the soil sub-surface, requires objective function to be defined and optimized. The objective function is defined as an error function which is minimum for the actual soil parameters. The accuracy and efficiency of the GPR system depends on the accuracy of the forward modeling and accuracy and efficiency of the inverse modeling. In this thesis work the common reflection coefficient method is explored for forward modeling and a spectral domain inversion technique is implemented for a layered ground surface. A three layered synthetic model of ground media is defined. The objective function is defined as an error function between the actual reflection coefficient and the modeled reflection coefficient. The variation of the objective function for different soil parameters like permittivity, height, conductivity is studied. Finally soil parameters are extracted with good accuracy by using global search techniques like Pattern Search and GA combined with Nelder-Mead Simplex method. It is also observed that the step by step detection of few parameters of first and second layer based on the properties of the reflection coefficient spectrum enhances the process of extracting complete electrical profile of the ground sub-surface.

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$\vec{D}$	Electric flux density
$\vec{E}$	Electric field intensity
$\vec{B}$	Magnetic flux density
$\vec{H}$	Magnetic Field intensity
$\vec{J}_c$	Conduction current density
$\vec{J}_d$	Displacement current density
ε_0	Electric permittivity of free space
μ_0	Magnetic permeability of free space
ε_r	Dielectric constant of medium
σ	Conductivity of medium

Chapter 1
INTRODUCTION

1.1 Introduction

Ground Penetrating Radar also called Surface Penetrating Radar is a high resolution geophysical technique [1] that uses the reflection mode of propagation of electromagnetic waves through the medium to map the subsurface [2]. It is an effective tool to detect metallic as well as non-metallic objects buried in the subsurface and also provide the pseudo image of the targets. With the advance in technology and data processing systems, it become a popular technique for solving environmental issues like detection of buried tank, detection of water level, detection of pipes, landmine detection etc.

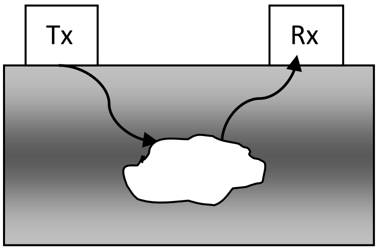

Figure 1.1: GPR works in reflection mode

The objective of this chapter is to provide an overview of the GPR system, its principle, data interpretation, types and various applications in brief of GPR today.

1.2 GPR Technology

It uses the principle of scattering of electromagnetic waves which is observed when a travelling EM wave encounters change in medium electric and magnetic properties while propagating from one medium to another. Transmitting antenna of the GPR system is designed to radiate the electromagnetic wave that travels through a material medium at a particular velocity which is a function of medium parameters like ϵ_r and μ. This dependency of the wave velocity on the medium parameters gives the basic idea of underground targets detection using GPR.

Wave velocity is given as:

$$v = {}^{c}/_{\sqrt{\varepsilon\mu}} \quad (1.1)$$

In GPR target detection the medium is generally considered as a non-magnetic medium. Therefore velocity of the electromagnetic wave is primarily determined by the relative permittivity of the medium. The wave spreads out from the transmitting antenna terminal and travel downwards until it encounters materials having different electrical properties than the surrounding medium. On hitting an object of different properties, an EM waves scatters and undergoes various phenomena like reflection, transmission, diffraction etc. A part of the electromagnetic energy reflected back to the ground is intercepted by the receiving antenna.

A GPR system consists of transmitter, receiver, signal processing unit and control unit. An antenna is a passive element which can be considered as a transducer which converts electric current on the metallic antenna element to electromagnetic wave energy if used at the transmitting end. It also converts EM waves to current in the antenna element by intercepting a part of the EM energy incident on it acting as a receiving antenna at the receiver end. If the same antenna is used for both transmission as well as reception of the EM signal, then the antenna system is called monostatic system. Whereas if the transmission and reception of GPR electromagnetic signal is achieved through different antenna system then it is called bistatic system.

A GPR system in time domain is always operated in bistatic mode. It is due to difficulty in design of fast switching circuits (duplexer) for changing the antenna operating in monostatic mode from transmitting to receiving element. Below diagram is an example of a bistatic configuration. Many number of reflection traces are stacked together to produce a reflection profile. GPR antennas are usually kept close to the ground so as allow good penetration of EM wave to the ground.

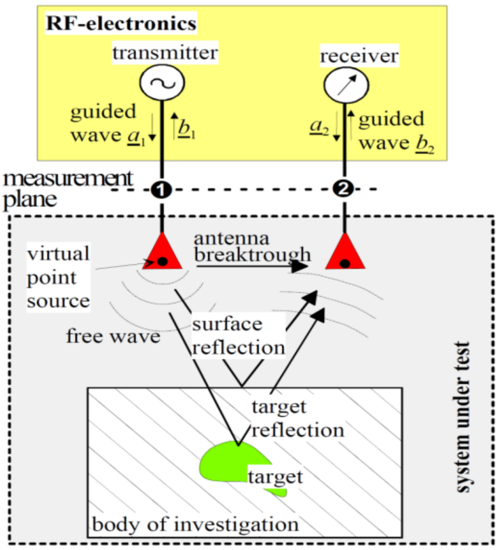

Figure 1.2: GPR working Principle [27]

1.3 Fundamentals of GPR

1.3.1 Electric and Magnetic Properties of medium

Electric properties of the subsurface medium is usually expressed in terms of permittivity (ε) and conductivity (σ) while permeability signify magnetic behaviour of the medium [9].

1.3.1.1 Permittivity – ε

It represents the ability of a material to store and release EM wave energy in the form of static electric charge particles. It can also be described as the ability of material medium to restrict the flow of charge carrier through it in the form of EM wave or the extent of polarization that medium undergoes due to electric field component of EM wave travelling through it. Hence EM wave velocity is function of ε . It is often expressed in terms of non-dimensional, relative permittivity ε_r, where

$$\varepsilon_r = {}^{\varepsilon}/_{\varepsilon_0} \tag{1.2}$$

It is generally a complex, frequency-dependent quantity with real part representing storage and imaginary part representing loss of EM wave energy. For approximate calculation of GPR wave velocity, it is generally simplified to its real constant low frequency component. It is given as: [4]

Permittivity, real part: $$\varepsilon'(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1+\omega^2\tau^2} \tag{1.3}$$

Permittivity, imaginary part: $$\varepsilon''(\omega) = (\varepsilon_s - \varepsilon_\infty) + \frac{\omega\tau}{1+\omega^2\tau^2} \tag{1.4}$$

1.3.1.2 Conductivity – σ

It is simply the ability of a material medium to allow passage of free electric charges through it under the influence of an electric field applied. Free charge carriers while flowing through the medium under effect of applied electric field causes attenuation and loss of energy in the form of heat. If conductivity value is low, wave suffers very small amount of attenuation while in highly conducting medium it is attenuated by large amount. So is the reason that GPR is ineffective in highly conducting medium like saline conditions and high clay contents. It is also complex in nature and increases with the frequency but considered as small or insignificant for radar frequency.

1.3.1.3 Permeability – μ

Practically the magnetic effect of materials like diamagnetic, paramagnetic phenomenon has insignificant effect on the propagating wave velocity of GPR and hence their magnetic permeability is taken as μ_0.

1.3.2 Forward Modeling

Forward modeling of a complex system can be stated in simple and general term as modeling of system components and parameters such that when supplied with a given input gives output closer to actual output received. Forward modeling of GPR requires following concepts and relations between the electric and magnetic components of travelling EM wave and electric and magnetic properties of medium.

1.3.2.1 Maxwell's Equations

It is partial differential equations written in integral or differential form that establish relation between the EM wave travelling through a medium and the medium electric and magnetic properties. It is general in the sense that it can be applied to homogeneous, inhomogeneous, linear, nonlinear and isotropic as well as nonisotropic media. Electric and Magnetic components of EM wave travelling can be obtained by solving Faraday's law or Ampere's Circuital law subjected to boundary conditions. It can be written mathematically in its differential form as shown below:

Gauss's Law of Electric Field: $$\nabla.\vec{D} = \rho \tag{1.5}$$

Gauss's Law of Magnetic Field: $$\nabla.\vec{B} = 0 \tag{1.6}$$

Faraday's Law: $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1.7}$$

Ampere's Circuital law: $$\nabla \times \vec{H} = \overrightarrow{J_c} + \overrightarrow{J_d} \tag{1.8}$$

1.3.2.2 Reflection Coefficient

Reflection of an EM wave occurs whenever there is impedance mismatching of the medium which is in turn due to irregularity of the medium electric and magnetic properties. Amount of incident wave reflected back to the original travelling medium is quantified by an EM parameter called ***'Reflection Coefficient'***.

It is defined as the ratio of reflected electric field to the incident electric field component of EM wave.

$$\Gamma = \frac{E_r}{E_i} \tag{1.9}$$

1.3.2.3 Transmission Coefficient

Whenever an EM wave encounters change in travelling media a portion of it gets reflected back allowing some portion to travel further which is called transmitted wave. The portion of incident wave travelling further is quantified by a physical parameter called Transmission Coefficient which is defined as the ratio of Transmitted wave to the incident EM wave at the interface of two medium. The wave is generally characterized by its electric field component.

$$\tau = \frac{E_t}{E_i} \tag{1.10}$$

1.3.3 Inverse Modeling

Inverse modeling is the process of getting system parameters when actual output and input to the system are known. The inversion method of layered medium can be categorized in two categories [10] depending upon the domain in which GPR is working:

i) Time Domain Inversion:

It is inversion of the GPR response when variable is time i.e. GPR is used in time domain. A pulse is transmitted to the ground, its attenuated version with addition of some noise or clutter is received by the receiving antenna. The pulse is processed further to get back the target range, material properties information [11]**.**

ii) Frequency Domain Inversion

In this method of inversion technique GPR is employed in frequency domain. Stepped Frequency CW GPR technique is often used by the researchers nowadays due to its various advantages over other frequency domain techniques. Frequency domain inversion requires forward modeling of GPR Subsurface system [12], and then a comparison between the actual received signal and the modeled one is made. The difference between two is minimized for actual target detection or medium parameters extraction.

While doing forward modeling of system, some assumptions regarding system geometry or system configuration are always taken to simplify the analysis. Due to the assumptions actual response of the system corresponding to a input always differs from modeled response for the same input. Hence inverse modeling requires the concept and application of Optimization theory to minimize the error between actual and modeled response. The error will be minimum if modeled system parameters approaches actual parameters of the system. The efficiency and accuracy of inverse modeling directly depends upon the correctness of forward modeling and efficiency, accuracy of inversion process.

1.3.3.1 Optimization Theory

Optimization is the process of obtaining best possible solution in given circumstances. It has application in almost every field of engineering like design, construction etc. It is also used in nonengineering field like taking managerial decision. It is actually finding the parameter that maximizes or minimizes a given function satisfying some predefined conditions. It generally means minimization as minimizing $f(x)$ is same as maximising $-f(x)$. The function to be minimized is called **Objective function.**

An Optimization problem can be classified in number of ways as discussed below:

i) Based on existence of constraints in the problem

a) Constrained Optimization

b) Unconstrained Optimization

ii) Based on the nature of design variable i.e. constant or varying with some other parameters

a) Static Optimization

b) Dynamic Optimization

iii) Classification based on nature of equations involved

a) Linear programming problem

b) Non Linear programming problem

c) Geometric programming problem

d) Quadratic programming problem

iv) Based on the nature of possible values of the design variable

a) Integer programming problem

b) Real-valued programming problem

v) Based on the deterministic nature of variable involved

a) Deterministic programming problem

b) Stochastic programming problem

Local minimum

A function of single variable $f(x)$ is said to have relative or local minimum at $\boldsymbol{x} = \boldsymbol{x}^*$ if

$f(x^*) \leq f(x^* + h)$, for all sufficiently small positive and negative value of h. On the other hand $if\, f(x^*) \geq f(x^* + h)$ for all values of h sufficiently close to zero, then $f(x)$ is said to have local maxima atx^*.

Global minimum

A function $f(x)$ is said to have a global or absolute minimum at x^* if $f(x^*) \leq f(x)$ for all values of x over which the function $f(x)$ is defined. Similarly a point x^* will be global maximum of $f(x)$ if $f(x^*) \geq f(x)$ for all values of x in domain of$f(x)$.

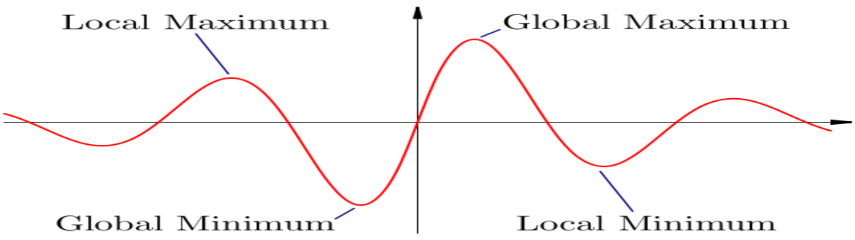

Figure 1.3 Local and Global minima

1.3.3.2 Optimization Techniques

It can be categorized as:

i) Direct Search method: It is based on objective function evaluation itself. In direct search method of optimization calculation of derivative of function is not required. It is also called non gradients methods. Ex- Nelder-Mead Simplex, Global multilevel coordinate search method.

ii) Descent method: Calculation of derivative of function is required in addition to function value evaluation. It is more efficient as compared to direct search method .Ex- Steepest descent method, Newton's method.

iii) Analytical Method: It is also called Differential Calculus method .In analytical method of optimization optimal value of decision variable is calculated first then optimal value of objective function is calculated. Classical method of optimization is an analytical method of optimization to find optimum of continuous and differentiable function.

iv) Numerical method: In numerical method of optimization, value of the objective function is first found at various combinations of the variables then optimal solution is concluded from these values. Ex- Simplex Method.

Need for Numerical method of optimization:

If the objective function is not an explicit function of the design variable such that manipulation of function with respect to its variable is difficult and we cannot invert the function to get back the design variable. But it is always possible to calculate the function value at various combinations of design variables. Hence we go for numerical method of optimization.

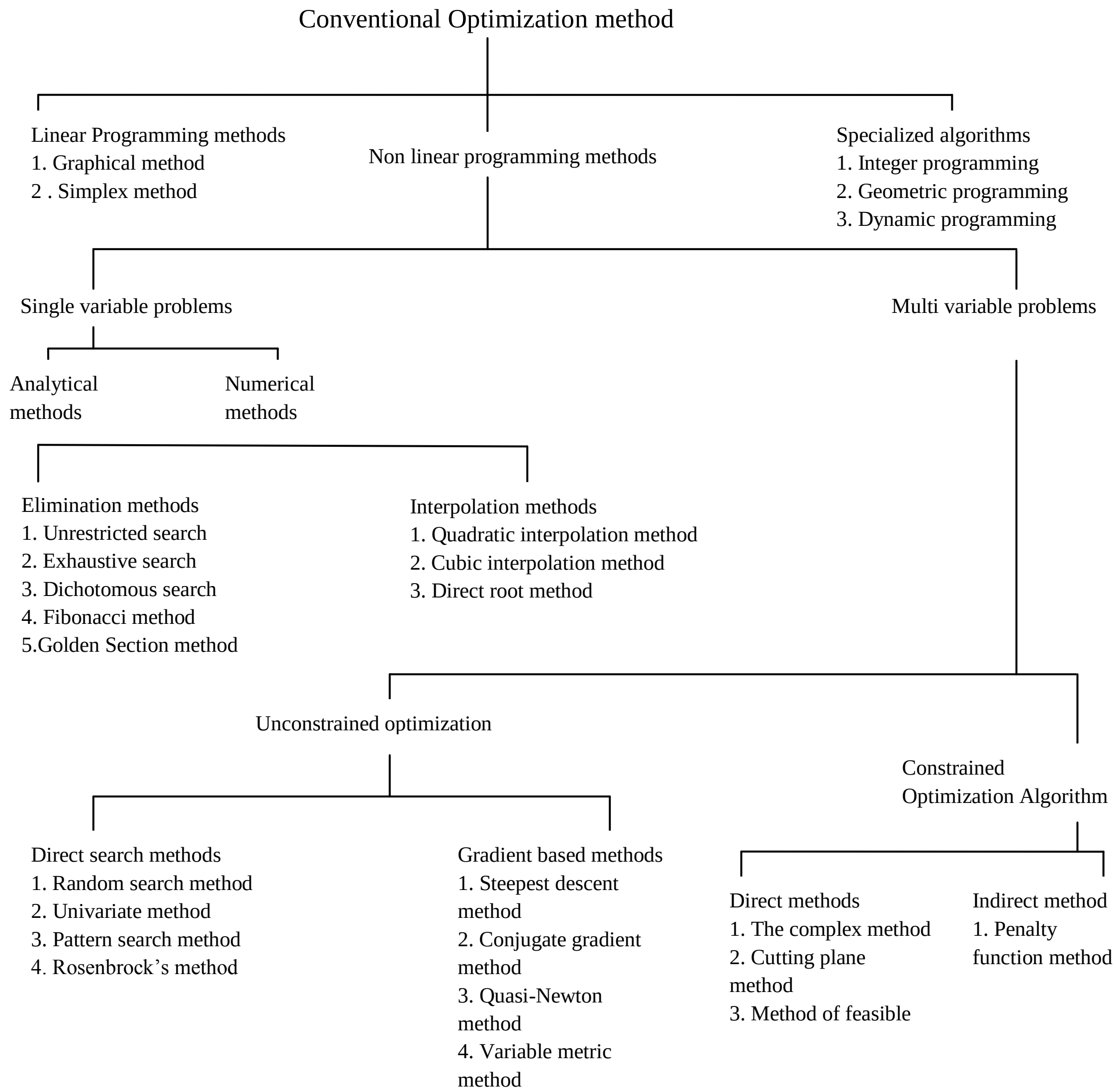

Figure 1.4 Classical Optimization Techniques

Merits and Demerits of Classical or traditional method of optimization:

Merits:

1. They are simple and easy to implements.

2. They have good convergence speed for local minima.

Demerits:

1. The solution of optimization problem using classical methods depends on the randomly chosen initial parameters vector. If initial guess lies in local basin, it will get stuck at local optimum and cannot give optimum solution.

2. Gradient based method of optimization cannot be used for a discontinuous objective function.

3. A traditional method of optimization cannot give solution to a variety of problem. However it is good in some cases having only local minima. Hence for problems having nonlinear variation of objective function with the design variables some other robust techniques are required to be developed and used for global optimization.

1.4 GPR Data Representation

Data recorded by a GPR are generally represented as one, two or three dimensional data set denominated by terminology A-scan, B-scan and C-scan.

A Scan

It is representation of a single waveform $f(x_i, y_j, t)$ recorded by a GPR with the antenna situated at a fixed position(x_i, y_j). In this case time t is the only variable which is related to depth of the target by the propagation velocity of the electromagnetic wave in the medium.

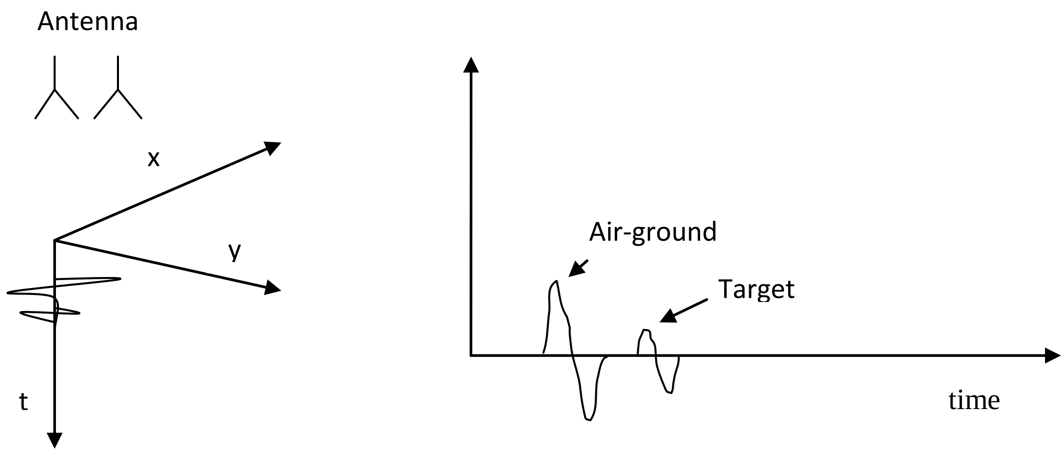

Figure 1.5: A Scan representations

B Scan

It is a two dimensional data set $f(x, y_j, t)$ obtained by moving the GPR along x-axis. It is collection of multiple A scan. When the amplitude of the signal is represented by a color scale, a 2D image is obtained. It is also called 2D scan.

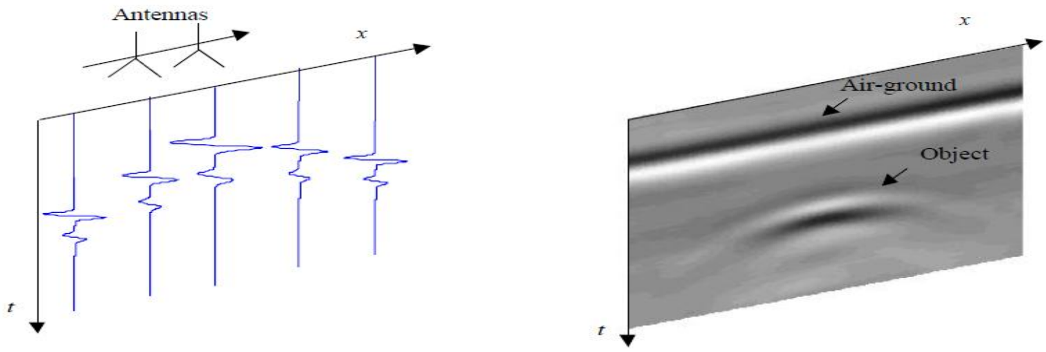

Figure 1.6: B Scan representations

C Scan

It is collection of multiple B scan. It is obtained by moving antenna over two dimensional area and is represented as a two dimensional image varying its amplitude with respect to time. GPR data obtained can be processed into 2-D, 3-D image visualization using software like EKKO_Mapper, EKKO_Project, EKKO_View, EKKO_Maper 3-D visualization, IcePicker, ConquestView etc.

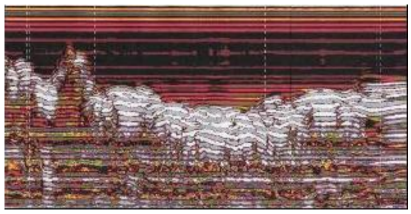

Figure 1.7: C scan representation [28]

1.5 GPR Classification

Depending on the manner in which data are acquired, GPR can be designed as time domain GPR and frequency domain GPR.

In time domain, data received is function of time interval, a time pulse of short duration at some pulse repetition frequency is sent to the ground and a backscattered pulse corresponding to the transmitted pulse is intercepted by receiving antenna.

In frequency domain, data recorded varies according to the frequency used for transmission. Frequency can be transmitted continuously or in discrete steps with continuity over discrete interval for the frequency band used for GPR application.

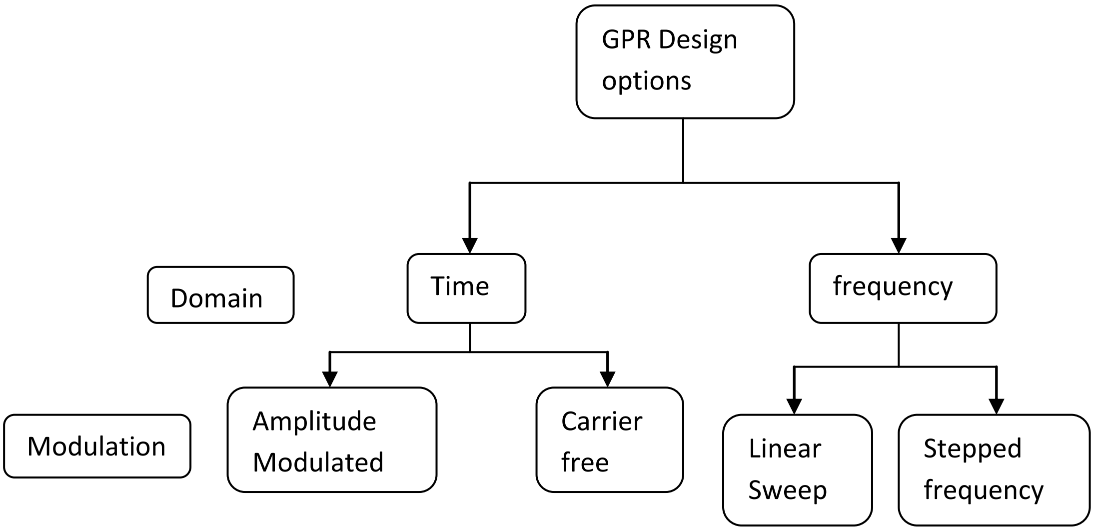

Figure 1.8: GPR system classification

1.6 Stepped Frequency Technique

It is a continuous wave radar technique since wave energy is transmitted and received continuously. It consists of Radio frequency source, receiver and digital signal processor (DSP). The source is allowed to step between a start frequency f_0 and a stop frequency f_{N-1} in equal and linear increments, where N is total number of increment value of frequency. Hence in SFCW technique [4], frequency is divided in number of steps over the band of operation.

Each segment is transmitted continuously, while transmission is discrete for the overall bandwidth. Hence a narrowband coherent receiver can be used for GPR reflected wave energy reception. By heterodyning a port of the transmitted signal with received signal, a composite signal called return signal is formed which is digitized for each interval and stored in discrete form for further processing. When a full sweep of N steps is complete, a frequency domain tool called Inverse Discrete Fourier transform is operated to change the collected discrete data from frequency domain to time domain. This inverse operation gives a time domain synthesized pulse. It has improved dynamic range [6].

Target range information is contained in ***'time of flight'*** of the wave, which is actually a phase path difference measurement. If target is closer to the GPR system, smaller phase change between the transmitted and received signal is observed as the travelling path of wave energy is shorter. But if the target is sufficiently apart, a larger phase change is observed due to longer propagation path.

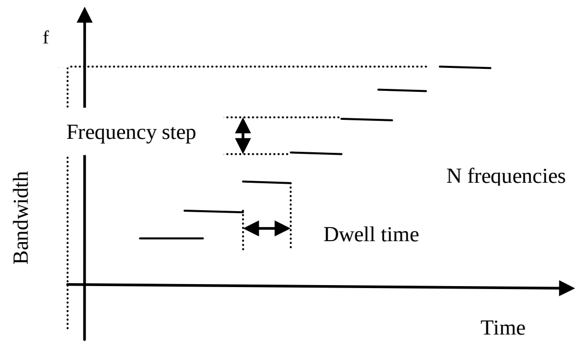

Figure 1.9: Stepped frequency continuous wave GPR

The amplitude of the EM signal received is a function of the radar cross section of the target, the range i.e. height below the ground level (vertical range) and the propagation loss of the ground.

A general SFCW GPR can be represented in simplified form as shown in the figure below:

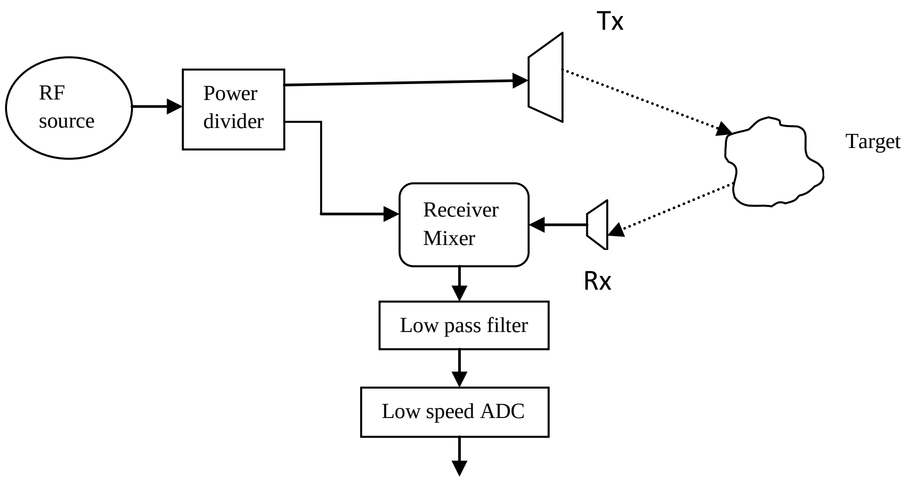

Figure 1.10: SFCW block diagram [4]

1.7 GPR Applications

With the advance in GPR technology and simultaneous improvement in its computing system due to fast processing computers, it is employed in various fields like:

i) Mining and Tunnelling

GPR can detect changes in rock types and find application in defining geological structure, mine site evaluation, tunneling design, mineral exploration etc.

ii) Forensics and Archaeology

GPR can be used to uncover buried caches drugs, money, weapons as well as to locate unmarked graves. Since GPR can detect water content of subsurface hence can be used to map historical sites, to define road and building locations.

iii) Locating Pipes and Cables

Since GPR is having capability to detect metallic as well as non-metallic structures, it can be used to locate buried pipes and cables.

iv) Military and Security

Due to unique property of GPR to detect metallic as well as non-metallic structures, it can be used in search and rescue tunnel location, landmine detection etc.

v) Agriculture and Forestry

Due to GPR's sensitivity to water content and change in material composition, it can be used in monitoring soil moisture content, mapping of drainage and irrigation, to monitor health of living tree.

vi) Ice and Snow

GPR can be used for snow depth monitoring for ski slope management, ice thickness for winter road safety, location of avalanche victims and for glaciological and polar ice-cap research.

Advantage of GPR System:

i) Fast data acquisition capability.

ii) High resolution system.

iii) Works well for metallic as well as non-metallic target environment.

Drawbacks:

i) Complex nature of data received.

ii) Gives hyperbolic image of the target.

iii) Data interpretation is an erroneous and difficult task and needs expertise and deep understanding of the knowledge in the field

1.8 Problem areas in GPR

GPR systems are similar to conventional radar systems in the sense that both measure target range i.e. radial distance of the target from the system irrespective of the direction by determining the two way travel time of an electromagnetic wave. Practically, however GPR systems are more complicated than conventional (ordinary) radar systems due to some unique problems associated with transmitting and receiving Electromagnetic energy through a subsurface medium. The main technical challenges in design and application of a Ground Penetrating Radar are:

i) Subsurface medium i.e. earth is typically inhomogeneous.

Inhomogeneous means the medium properties likeσ, ϵ_r extent of media varies from point to point due to varying composition like sand, water, air and other mineral deposits of underground media. Hence the velocity of an EM wave which is function of the medium properties also varies dramatically from point to point and is unknown initially due to unknown medium. Hence the complete analysis of unknown subsurface and hence detecting the target is a time consuming and rigorous process.

In ordinary radar system travelling media is generally air through which the velocity of propagation of an electromagnetic wave is known. Hence target range can be easily calculated by determining two way travel time of the electromagnetic energy.

ii) Poor wave penetration in the subsurface medium

Some ground media like wet clay, salt water is good absorbent of EM wave at the frequency band of operation of GPR. Hence penetration through the subsurface is very poor and is not as good as in air medium.

The wave gets attenuated while propagating through a medium due to the medium properties called conductivity and attenuation and hence penetration is directly proportional to the frequency of travelling EM wave .Hence low frequency yields greater subsurface penetration. Unfortunately, lower frequency results in decreased target range resolution which depends in inverse manner on the system bandwidth i.e. frequency of operation. So GPR system needs to be properly designed to establish a tradeoff between the two factors depending upon the application.

iii) **High level of clutters**

Often GPR antenna is operated very close to the ground surface. Sometimes antenna mouth touches the ground to improve the penetration depth. The field pattern in the medium can be very different from the one in air because of the proximity effect. This can increase the clutter level of GPR system due to multiple reflections at various interfaces. Moreover ground surface roughness is stochastic with respect to location and time. This in turn introduces interferences in form randomness on the measured EM field.

1.9 Motivation and Objective of the thesis

GPR is a high resolution, nondestructive electromagnetic technique to detect subsurface targets. Its application area has increased over the years with advance in computing and data processing systems and hence researchers are more focused in GPR technology advancement. It can detect both metallic as well as non-metallic targets and this makes it versatile in nature from application point of view.

The main aim of this thesis work is:

- To study different modeling techniques available for forward modeling of GPR system.
- To find a modeling scheme to represent the GPR signal in the complex ground media correctly.
- To extract the subsurface parameters by a suitable inverse modeling scheme efficiently.

1.10 Organisation of the Thesis

The complete thesis is divided into different chapter depending upon the idea or information it is sharing. Each chapter starts with introduction and concludes with the idea at its end.

Chapter 1: It has discussed GPR principle, its types, data representation methods and various applications in brief. It has highlighted some of the dominating advantages of frequency domain GPR on time domain GPR and hence has discussed SFCW GPR in detail.

Chapter 2: This chapter gives a brief overview of the forward modeling method available for GPR. It discusses material electric and magnetic parameters and optimization theory in brief. It also discusses the modeling method adopted in this project work.

Chapter 3: This chapter is dedicated for discussion of method and results of synthetic modeling approach adopted for GPR subsurface-system in this project work. It also discusses the steps taken and result obtained after spectral domain inversion is implemented.

Chapter 4: This chapter covers summary and conclusion of the whole thesis.

Chapter 5: This thesis ends with chapter 5 which includes all the references taken during project work and writing of this thesis.

Chapter 2

MODELING OF GROUND PENETRATING RADAR

2.1 Introduction

GPR system is similar to a conventional radar system as both uses electromagnetic principle to extract the attributes of target. But they differ in terms of application, system designing parameters and system limitations as they are having different travel medium having different properties governing EM wave propagation. Hence a GPR system is specially modeled and designed to have proper bandwidth and system configuration which varies according to the application.

GPR modeling is actually a three stage process consisting of analysis, synthesis and optimization. Data gathered by the modeling process is testified on field trial basis to validate the model suggested for the GPR system. It is actually done to know the behavior of GPR system response with respect to ground properties so that an unknown target can be detected and classified. The GPR response has nonlinear relation with the subsurface parameters hence the error between actual response and modeled response needs to be optimized to get the correct parameters. There are several analytical and numerical model suggested by the researchers in their research paper for GPR system modeling. This chapter will discuss some of the analytical modeling techniques adopted for SFCW GPR system. It will also give a brief introduction to the optimization techniques generally applied for GPR inverse modeling.

2.2 Literature Survey

Forward modeling of GPR subsurface system is an important aspect in parameter extraction since accuracy and efficiency of inverse modeling depends directly upon the effectiveness and correctness of forward model suggested. There are many analytical and numerical approaches taken by GPR researchers for forward modeling of the system depending upon the application.

2.2.1 Analytical Modeling

In analysis of responses, several assumptions are taken to simplify the problem to the level of analysis. The valid approximation and assumptions makes GPR data processing and hence target identification fast and accurate. Some of the analytical modeling techniques are: Standard surface reflection Coefficient method, CMP method, Ground Wave propagation method, transmission line method.

Standard surface reflection Coefficient method is the easiest method generally applied for GPR having target in its far field region. The reflection coefficient for normal incidence of EM wave at the interface separating two different medium, can be expressed in terms of medium electric properties as discussed later in the chapter. This forms the basis for the forward modeling using reflection coefficient method. Lambot [5] has estimated water content of soil using this method and compared the result with that obtained using full wave inversion method. The two results match to good extent. The method is also adopted successfully by Huang Zhonglai and Zhang jianzhong [10] for highway pavement quality estimation.

GPR signal can be analyzed using ray-tracing based techniques and tomographic inversion method [24, 25]. **CMP (Common Mid Point) method** can be used to estimate permittivity value governing EM wave propagation velocity [8]. The method is adopted by Nakashima et al.[26] to estimate ground water level in an environment with multiple reflection taken from different depth. The vertical permittivity is then estimated from interval velocity using CMP. It is not practically efficient as multiple numbers of measurements have to be taken for single profile estimation and data estimated is prone to high uncertainty unless clear reflecting surfaces are present in the ground.

Ground Wave is defined as the wave directly travelling from the transmitting to the receiving antenna of GPR travelling through few centimeters of the soil. It can be identified by taking multiple measurements with different antenna separations. Its velocity can be estimated by Single trace analysis. However, higher uncertainty in the wave velocity measured is observed by *Huisman et al.* [27] and he suggested GPR taking multiple measurements with multiple receivers for mapping applications. The wave velocity determined is characterized by vertical heterogeneity of the soil. Due to few centimeters soil characterization, its application is restricted to agriculture as crop root develop mainly in first soil meters.

Transmission line modeling approach involves replacement of each soil layer by equivalent impedance. The different travelling wave parameters i.e. attenuation constant, propagation constant, wave impedance is calculated for each layer and hence reflection coefficient, transmission coefficient for multilayer geometry is formulated with layer properties.

Full wave inversion method is based upon the analysis of basic equations called **Maxwell's equations** giving the relation between EM wave components and travelling medium parameters. It involves 3-D modeling of ground layers [7]. It is a complicated and challenging job for researchers. But its application grows as the analysis of 3-D mathematical equations and relations can be easily done now with superfast data processing computer embedded with high technology microprocessor. Full wave modeling approach is applied by Lambot [28] to measure soil hydraulic properties. After success of inversion method, its application is extended to soil miniaturization estimation, mine detection etc.

2.2.2 Numerical Modeling Technique

It is numerical technique requiring very small part of analysis. It can take care of boundary conditions and very fewer assumptions are involved regarding the media and target geometry. Hence is more efficient as compared to analytical modeling in the complex situations where analysis of subsurface geometry and EM wave equations is not possible. Some of the popular numerical modeling techniques adopted by GPR researchers are FDTD, FEM and MOM.

FDTD is a numerical technique to solve Maxwell's equations. The time varying continuous Maxwell's equations, originally in partial differential form is discretized using central difference equations to the space and time partial derivatives and implemented in software. The equations are solved by solving electric and magnetic field in a volume of space at a given instant of time in subsequent manner until the complete model is solved. It is a useful method to model EM wave propagating through complex media like earth. It is an ideal tool for modeling transient EM fields in inhomogeneous media like soil as it fit relatively into finite difference grid and absorbing boundary conditions can truncate the grid to simulate an infinite region **.**FDTD modeling [8] is used by many authors for the simplified antenna mostly dipole system. Since temporal as well as spatial discretization is involved which may leads to requirement of excessive memory space of the data acquisition and data processing systems.

2.2.3 Optimization Schemes in GPR

Inverse modeling is a process of extracting unknown target parameters and hence identifying its properties from the GPR response i.e. reflected wave energy. Inverse modeling requires optimization of objective function defined according to forward modeling approach. Generally objective function defined for GPR is having nonlinear relationship with the design variable i.e. subsurface parameters. This causes occurrence of multiple local minima whose global point actually represents the design variable values. Depending upon the nature of objective function several global optimization techniques are presented by GPR researchers in their papers for model inversion.

Hunang Zhonglai and Zhang jianzhong [10] has adopted Stochastic Hill Climbing algorithm for model inversion of parameters by Spectral domain inversion technique. Since random initiation of start vector can take more time while converging hence he has presented technique of step by step inversion to increase the conversion speed. The parameters inverted are found to be closer to the actual value and hence result is quite promising.

Lambot *et al* [7] has tried hybrid approach in his paper to increase the convergence speed of model inversion. He has applied GMCS (Global Multilevel Coordinate Search) algorithm with a local minimizer Nelder Mead Simplex algorithm for model inversion. It is found that if a global optimization technique which is used to locate global basin is combined with a local optimizer, convergence speed and accuracy of the parameters extracted shows a remarkable improvement over the approach where a single global technique is used for inversion.

Some of the optimization techniques generally employed in GPR inverse modeling includes Nelder-Mead Simplex method, Pattern search method, Simulated Annealing method, Genetic Algorithms, Global Multi Level Coordinate Search method, Stochastic Hill Climbing method.

Pattern Search Algorithm

It is a direct search method of optimization. In Univariate method of optimization, search direction is always along the parameters axis, hence it is slow in convergence. In some cases it may not converge at all. Hence direction of search is randomize in pattern search method which takes a univariate step equals to number of variables and searches for the minimum point along direction defined as:

$$S_i = X_i - X_{i-m} \tag{2.1}$$

Where, X_i = point obtained at the end of univariate step.

X_{i-m}= starting point before taking m univariate step.

Nelder-Mead Simplex Algorithm

Simplex is a geometric figure formed by a set of n+1points in n-dimensional space. It is a direct search method of optimization. The basic idea in the simplex method is to compare the function value of objective function at the n+1 vertices of a general simplex and move the simplex gradually towards optimum point i.e. minimum function value point during the iterative process. The movement of the simplex is achieved by three operations: Reflection, Contraction and Expansion.

The reflected point, $$X_r = (1+\alpha)X_o - \alpha X_h \tag{2.2}$$

Where, X_h = vertex corresponding to maximum function value.

$$X_o = \frac{1}{n}\sum_{i=1,\neq h}^{n+1} X_i \tag{2.3}$$

α= reflection coefficient, > 0.

The expansion point is given as: $$X_e = \gamma X_r + (1-\gamma)X_o \tag{2.4}$$

$\gamma =$ Expansion coefficient, > 1.

The contraction point can be given as: $$X_c = \beta X_h + (1-\beta)X_o \tag{2.5}$$

β= contraction coefficient (0≤ $\beta \leq 1$)

Genetic Algorithm (GA)

It is heuristic search method based on the concept of natural evolution. It is a kind of evolutionary algorithm i.e. an algorithm inspired by biological evolution such as inheritance, mutation, recombination and selection. It is used in various fields like engineering, chemistry, mathematics, physics, computational science etc.

In a genetic algorithm, evolution of a population of candidate solution to an optimization problem takes place which leads to better solution. Since each candidates has its own unique properties which can be mutated and altered giving a set of new candidates. The solution is encoded generally in binary i.e. in terms of '0' and '1' format. It is an iterative procedure.

The evolution starts with a population of randomly generated candidates. In each generation the fitness of an individual in the population is evaluated; the fitness can be given as the value of an objective function to be minimized. The more fit individuals are stochastically selected from the current population and its properties are modified to form new individuals which are carried as candidates for the next iteration. The stopping criteria or the convergence condition of the algorithm can be specified in terms of maximum number of iterations or the minimum value of the objective function.

2.3 Spectral Domain Inversion Method

In this Project work frequency domain inversion method is adopted i.e. Stepped Frequency Continuous Wave GPR configuration is used. Spectral domain inversion requires forward modeling of the GPR subsurface system. Hence **Standard Surface Reflection Coefficient method** is employed for forward modeling of subsurface system.

The modeling works with the following assumptions:

i) The antenna is situated off the ground and hence target is located in the far field region of antenna. Hence the wave propagating through the medium can be thought of plane wave.

ii) The soil is either nonconductive or having very small value of conductivity.

iii) The medium through which EM wave is travelling is nonmagnetic in nature.

iv) The antenna distortions effects are negligible.

v) At each separating interface, EM wave is considered to be normally incident.

With the following assumptions, analysis of multilayered subsurface is done. I have tried to follow and verify a forward modeling approach proposed by Huang zhonglai [10] during my project work**.** He has presented a frequency domain inversion method based on the reflection coefficient forward modeling approach to obtain position, permittivity and conductivity of the different layer.

An L-layered model of soil surface in considered as shown in fig (2.1). Each layered section is assumed to be **linear, homogeneous and isotropic** and also the physical properties $(\varepsilon, \mu, \sigma)$ are considered as independent of frequency.

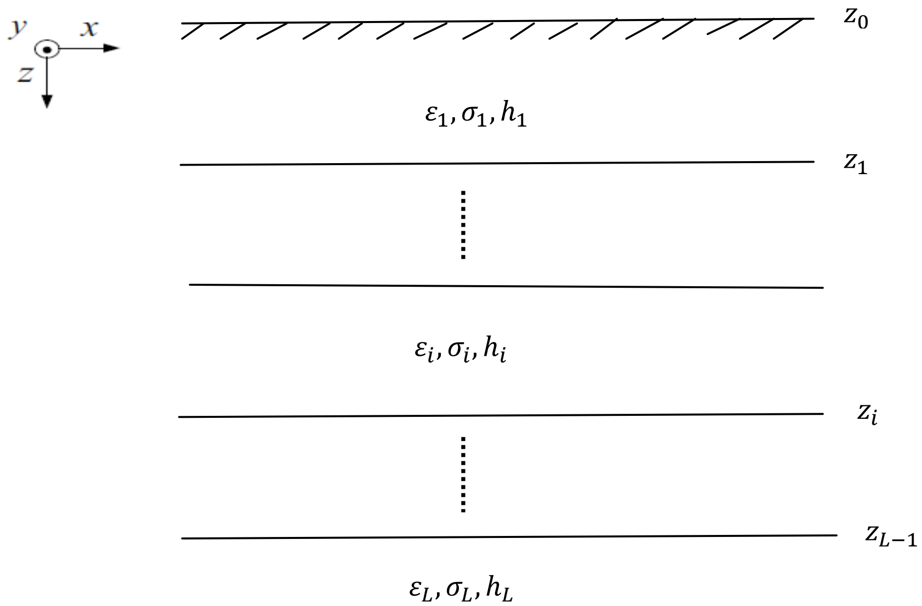

Figure 2.1 L layered model of sub surfaces

If w (t) is the transmitted wave, then its attenuation version with some noise added will be received at receiver end. Due to pre assumption of linearity of the soil medium received wave at the antenna terminal will be superposition of L reflected wave energy corresponding to L interfaces [10].

$$s(t) = \sum_i A_i \, w(t - t_i) + n(t) \qquad (2.6)$$

Where A_i and t_i are amplitude and time delay respectively of the backscattered energy due to i-th interface, n (t) is noise. S(t) is received wave and w (t) is pulse transmitted.

Incident wave is assumed to be normal to the interface directed towards x direction and propagating in z direction. The equation governing wave propagation for the above mentioned conditions can be obtained from the Maxwell's equations discussed in Chapter 1 and it is given as:

$$\frac{\partial^2 E_x(z)}{\partial z^2} = \gamma^2 E_x(z) \tag{2.7}$$

On further solving equation (2.7) , a plane wave propagating equation in a general charge free lossy region along positive z direction having only x component can be given as:

$$E_x(z) = E_{x0} e^{-\gamma z} \tag{2.8}$$

$$= E_{x0} e^{-\alpha z} \cos(\omega t - \beta z)$$

Where E_{x0} is maximum amplitude, z is the distance travelled by EM wave and γ is propagation constant in terms of m^{-1} , given as:

$$\gamma = \alpha + j\beta \tag{2.9}$$

Where α is attenuation constant in terms of Neper/meter, β is phase constant in terms of radian/meter.

They are given by the following equations:

$$\begin{cases} \alpha = \omega \sqrt{\frac{\varepsilon\mu}{2} [\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1]} \\ \beta = \omega \sqrt{\frac{\mu\varepsilon}{2} [\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1]} \end{cases} \tag{2.10}$$

Where ω is the angular frequency of EM wave; ε is the relative permittivity of medium; σ is the electric conductivity of medium and μ is the magnetic permeability of medium.

For single interface with normal incidence of wave, reflection coefficient is given as:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \tag{2.11}$$

Where η is the wave impedance of corresponding medium. For a lossless medium having negligible value of conductivity, it is given as

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \tag{2.12}$$

Since medium is considered as nonmagnetic in nature hence magnetic permeability is constant value independent of nature of medium and wave impedance is only function of medium relative permeability. Hence reflection coefficient for non-magnetic medium is given as:

$$\Gamma = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \tag{2.13}$$

Since **ε is a real number for GPR application**, so reflection coefficient at an interface of medium 1and 2 is represented by symbol $\boldsymbol{r_{12}}$.

Transmission coefficient between nonmagnetic medium 1and 2 for normal incidence of EM wave are given as:

$$\tau_{12} = {2\varepsilon_1}\Big/{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \tag{2.14}$$

2.3.1 Global Reflection Coefficient Derivation

In case of multilayered medium, reflection coefficient expression defined in equation 2.13 cannot be applied as wave reaching receiving antenna terminal undergoes multiple number of reflection due to multiple reflecting surfaces. Above defined reflection coefficient gives relation between the reflected wave by an interface separating two different medium and an incident wave energy falling at the same interface of the medium.

Hence a more general term representing the relation between wave emitted by the transmitting antenna of GPR and wave reflected by any interface separating two different media is defined; it is called ***Global Reflection Coefficient.*** It is defined as the ratio of electric intensity of reflection wave at an interface and radar emission wave. Global reflection coefficient for the interface between medium 1 and 2 is denoted by the symbol$\tilde{\mathbf{r}}_{\mathbf{12}}$.

For derivation purpose, a three layer subsurface medium separated by two interfaces is considered

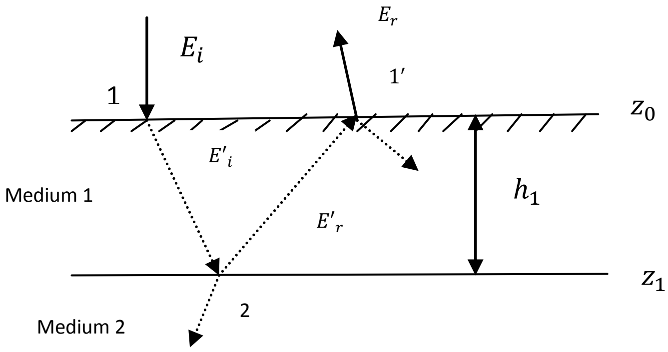

Figure 2.2 Three layers modeling of subsurface

Here a global reflection coefficient term is derived for z_0 interface and it is generalised further for any interface separating two different medium.

Global reflection Coefficient for z_1 interface (between medium 1 and medium 2) is defined as:

$$\tilde{\boldsymbol{r}}_{\mathbf{12}} = \frac{\boldsymbol{E_r}}{\boldsymbol{E_i}} \tag{2.15}$$

Here wave is assumed to be incident normally at each interface for deriving final global reflection coefficient expression, while above diagram consider actual transmission characteristics and hence depicts oblique incidence of the wave at an interface.

For node (2), $$\mathrm{r}_{12} = \frac{E'_r}{E'_i} \tag{2.16}$$

For node (1), $$\tau_{01} = \frac{E'_i}{E_i}$$

Hence $$E'_i = \tau_{01} E_i$$

Wave incident at node (1) travels a path of length equals to thickness of layer i.e. h_1. From equation (2.8), E'_i at node (2) is given as:

$$E'_i = \tau_{01} e^{-\gamma h_1} E_i \tag{2.17}$$

Combining equation (2.16) and equation (2.17)

$$E'_r = r_{12}\tau_{01}e^{-\gamma h_1}E_i$$

Hence $$\frac{E'_r}{E_i} = r_{12}\tau_{01}e^{-\gamma h_1} \tag{2.18}$$

For node (1′) $$\tau_{10} = \frac{E_r}{E'_{r(at\ node\ 1')}}$$

Hence $$E_r = \tau_{10}E'_r e^{-\alpha_1 h_1}e^{j\beta_1 h_1} \tag{2.19}$$

Using equation (2.9), (2.18) and equation (2.19)

$$\tilde{r}_{12} = r_{12}\tau_{01}\tau_{10}e^{-2\alpha_1 h_1} \tag{2.20}$$

Relation between transmission and reflection coefficient is given as:

$$\tau = 1 + r \tag{2.21}$$

Hence $$\tau_{01} = 1 + r_{01},\ \tau_{10} = 1 + r_{10},\ r_{10} = -r_{01}$$

So final expression for global reflection coefficient can be given as:

$$\tilde{r}_{12} = r_{12}(1 - {r_{01}}^2)e^{-2\alpha_1 h_1} \tag{2.22}$$

This expression can be generalized to find global reflection coefficient at i-th interface as:

$$\boldsymbol{\tilde{r}_{i,i+1} = r_{i,i+1}\prod_{k=0}^{i-1}(1 - {r_{k,k+1}}^2)\prod_{k=1}^{i}\exp(-2\alpha_k h_k)} \tag{2.23}$$

Where α_k is, attenuation of EM wave in k-th layer; h_k is distance travelled by EM wave in k-th layer or height of k-th layer.

Total Global reflection coefficient at the receiver antenna is obtained by superimposing global reflection coefficient produced by each interface taking into account the time delay of propagation.

2.4 Conclusion

Target detection by GPR is a complex task that involves forward and inverse modeling of the system. Out of various forward modeling techniques available and discussed in this thesis, a simple technique named Surface Reflection Coefficient modeling has adopted for further analysis and inversion. As explained clearly, classical definition and expression of reflection coefficient is not sufficient for analyzing multilayer subsurface. Hence a more general definition and expression for reflection coefficient called Global Reflection Coefficient is derived and presented.

Chapter 3
SYNTHETIC MODELING AND INVERSION OF GPR

3.1 Introduction

This chapter contents synthetic modeling of multilayered subsurface using standard reflection coefficient analytical modeling as discussed in Chapter 2. A general three layered geometry is considered for analysis. After completion of forward modeling, an objective function is defined to extract the subsurface parameters. Since each parameters affect EM wave properties in its unique way, hence behavior of the objective function with respect to subsurface parameters under consideration is studied which further suggest the optimization technique to be applied for inversion with improved accuracy and efficiency.

3.2 Synthetic Modeling of GPR Subsurface

A synthetic modeling of three layered medium consisting of air, soil and a general medium is done using the method proposed by Huang Zhonglai [10]. The layered medium can be represented as:

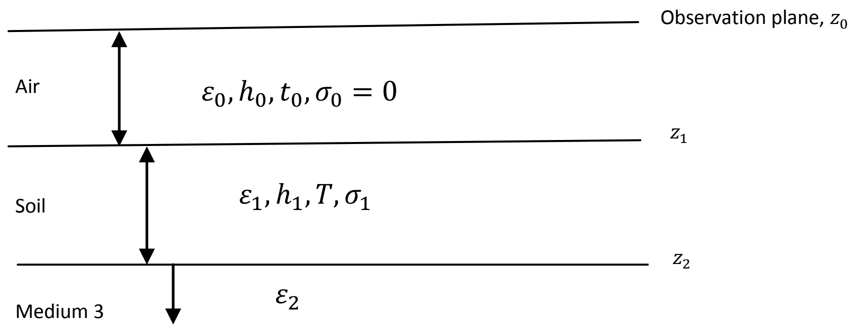

Figure 3.1 Three layered medium assumption of sub surface for synthetic modeling

T: two way travel time of EM wave in second medium i.e. soil.

t_0: Two way travel time of EM wave in first medium i.e. air.

Each layer is assumed to be linear, homogeneous and isotropic. Third layer is taken as an infinite layer such that wave entering through it gets highly attenuated and is not coming back to the ground. Hence net reflection coefficient measured at the observation plane is superposition of that obtained from first and second interface, each delayed by respective time delay path. It can be given mathematically as:

$$\tilde{\rho}(t) = \tilde{r}_1 \delta(t - t_0) + \tilde{r}_2 \delta(t - t_0 - T) \tag{3.1}$$

Where $\tilde{r}_1$ is the global reflection coefficient due to first interface and is equal to local reflection coefficient at the interface of medium 1 and 2, since wave is not attenuated while travelling through air. Hence

$$\tilde{r}_1 = r_{12} = \frac{\sqrt{\varepsilon_0}-\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_0}+\sqrt{\varepsilon_1}} \tag{3.2}$$

$\tilde{r}_2$ is the global reflection coefficient at the second interface and is given as:

$$\tilde{r}_2 = r_{23}(1 - r_{12}{}^2)\exp(-2\alpha_2 h_2) \tag{3.3}$$

Where $r_{23}$is the local reflection coefficient at the interface between 2nd and 3rd medium i.e. soil and an infinite medium and is given as:

$$r_{23} = \frac{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}} \tag{3.4}$$

Net Global Reflection Coefficient in frequency domain can be obtained by taking Fourier transform of equation (3.1) and it is given as:

$$\rho(f) = \tilde{\rho}(f)\exp[-i2\pi f\left(t_0 + {}^{T}\!/_{2}\right)] \tag{3.5}$$

Where, $$\tilde{\rho}(f) = 2r_e\cos(\pi f T) + i2r_o\sin(\pi f T) \tag{3.6}$$

$$r_e = (\tilde{r}_1 + \tilde{r}_2)/2$$

$$r_o = (\tilde{r}_1 - \tilde{r}_2)/2$$

3.3 Nature of Modeled Amplitude and Phase Spectrum

To plot the modeled amplitude and phase spectrum following synthetic values of parameters are taken:

	Parameters								
	f (MHz)	$\boldsymbol{h_0}$**(m)**	$\boldsymbol{h_1(m)}$	$\boldsymbol{\varepsilon_0}$	$\boldsymbol{\varepsilon_1}$	$\boldsymbol{\varepsilon_2}$	$\boldsymbol{\sigma_1}$**(mS/m)**	$\boldsymbol{t_0}$**(nsec)**	**T(nsec)**
Value taken	500-3500	0.4	0.2	1	2.4	4.4	0.02	2.67	2.07

Table 1: Parameter Values used in Simulation

Reflection Coefficient amplitude and Phase Spectrum was simulated for the three layer modeling of subsurface medium.

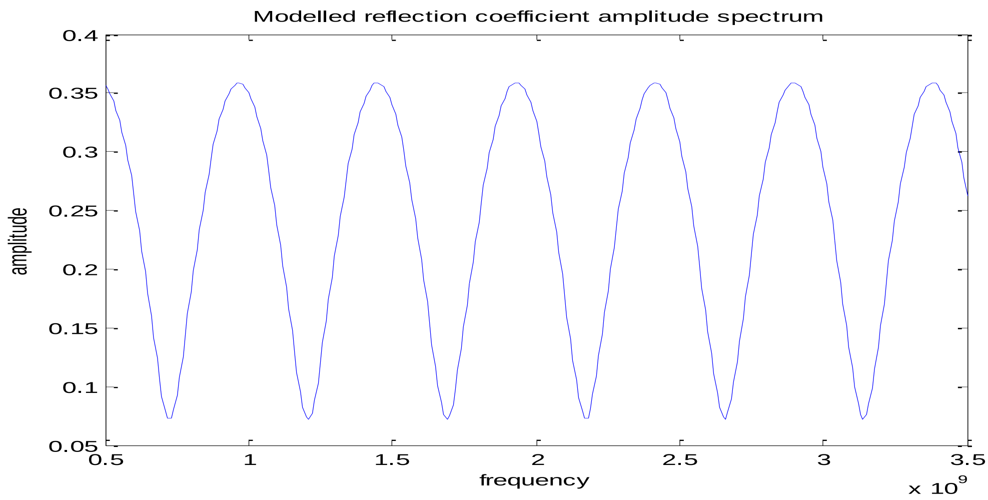

Figure 3.2 Modeled Reflection Coefficient Amplitude Spectrum

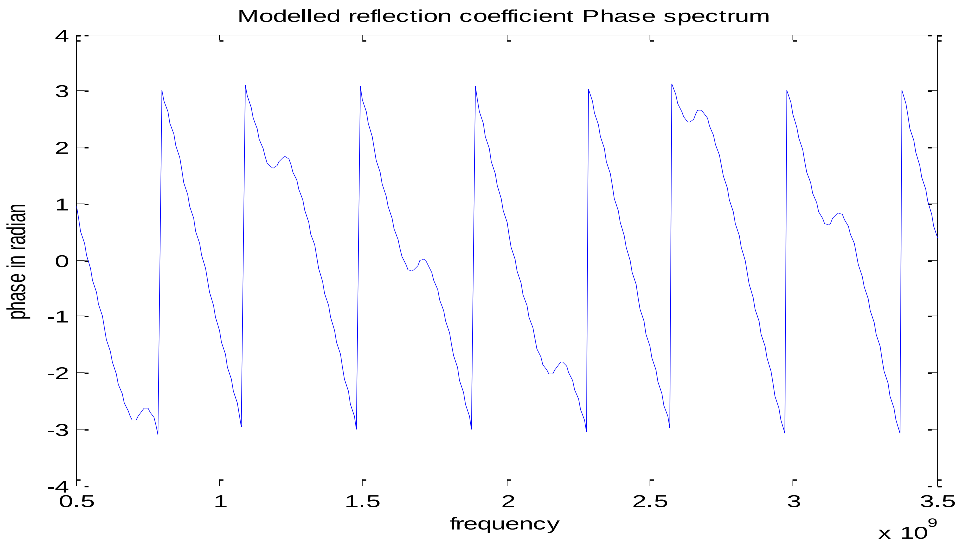

Figure 3.3 Modeled Reflection Coefficient Phase Spectrum

Reflection Coefficient amplitude as well as Phase varies periodically over the frequency band of operation for GPR.

3.4 Variation of Objective function with subsurface parameters

As already discussed in Chapter 2, optimization is an integral part of GPR system modeling. The function to be optimized is called an objective function. Here objective function is defined as an error function between actual and modeled reflection coefficient. Since medium parameters affect wave amplitude and phase value differently, hence error function has studied separately for reflection coefficient amplitude and phase value.

Objective function defined as an error function in terms of reflection coefficient amplitude is called amplitude error function and is given as:

$$O_b(\varepsilon, \sigma, T, t_0) = \sqrt{\frac{1}{f_H - f_L}\left(\sum_{f=f_L}^{f_H}(\rho_{modeled\ amplitude} - \rho_{actual\ amplitude})^2\right)} \quad (3.7)$$

3.4.1 Variation of amplitude error function with subsurface parameters

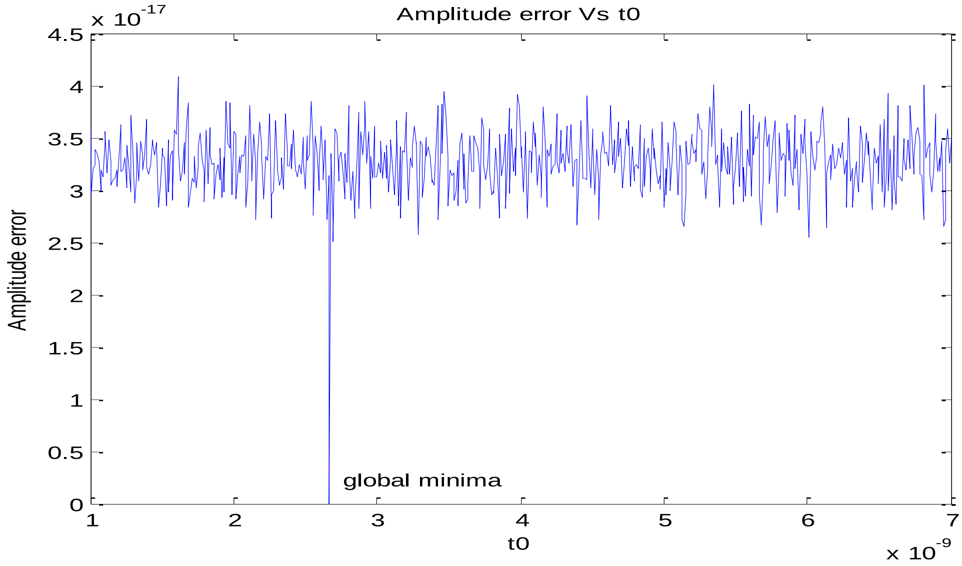

Figure 3.4 Variation of Amplitude error with respect to two way travel time of EM wave in air media

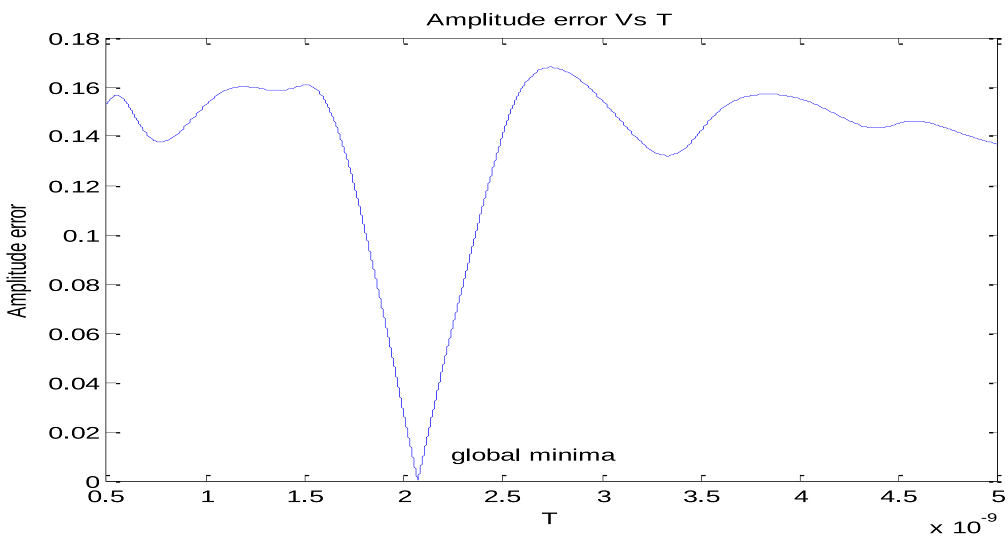

Figure 3.5 Variation of Amplitude error with respect to two way travel time of EM wave in soil media

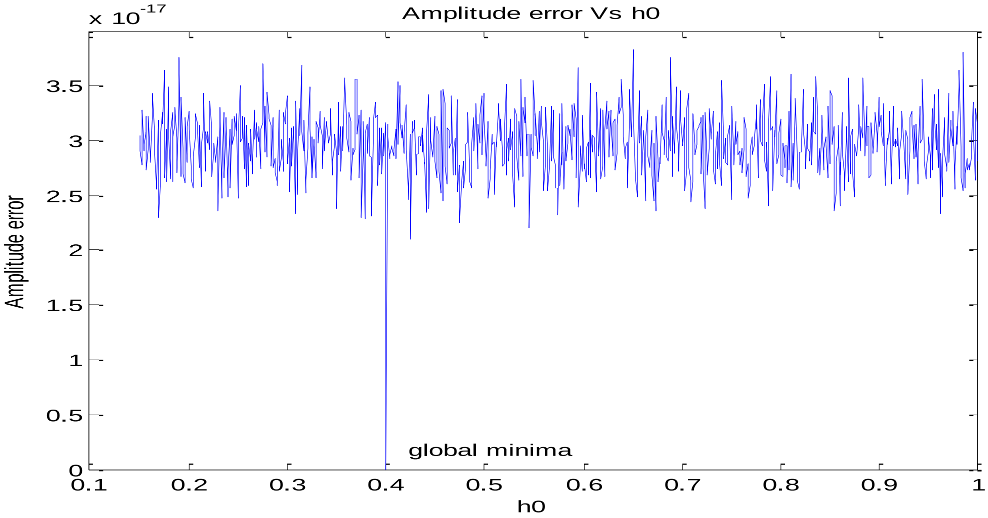

Figure 3.6 Variation of Amplitude error with respect to height of air media layer

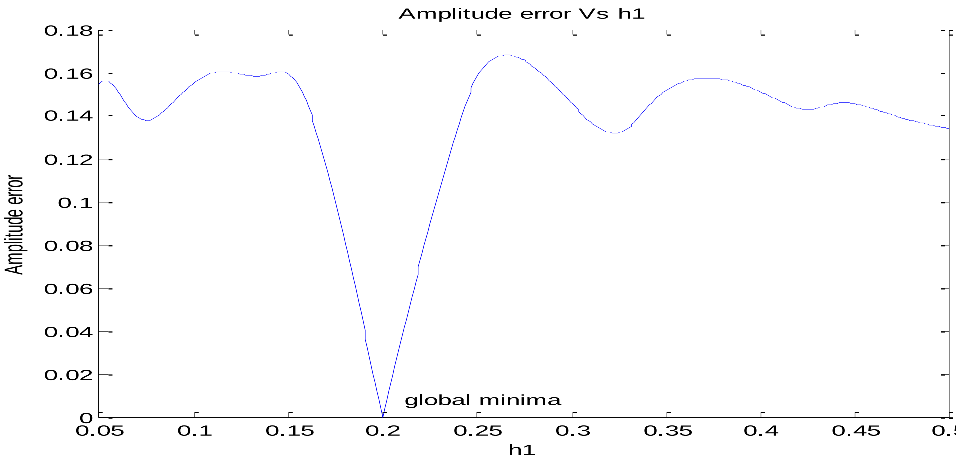

Figure 3.7 Variation of Amplitude error with respect to height of soil layer

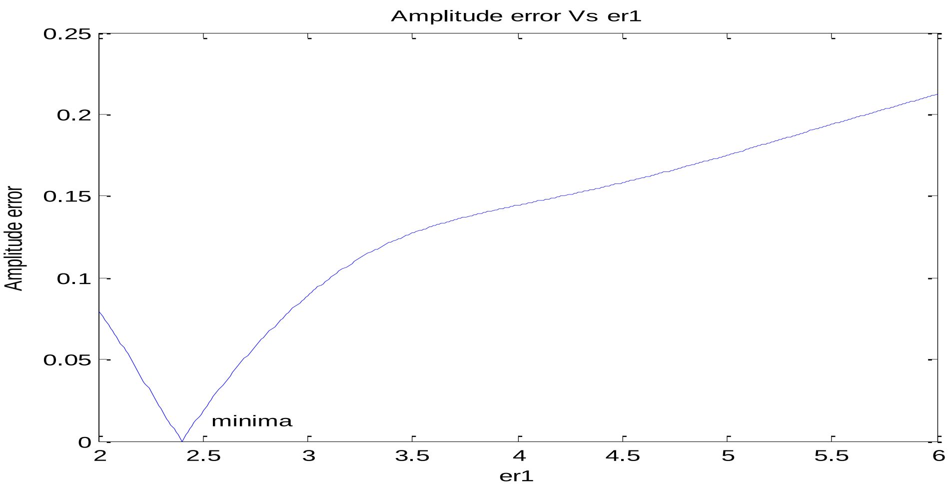

Figure 3.8 Variation of Amplitude error with respect to soil layer permittivity

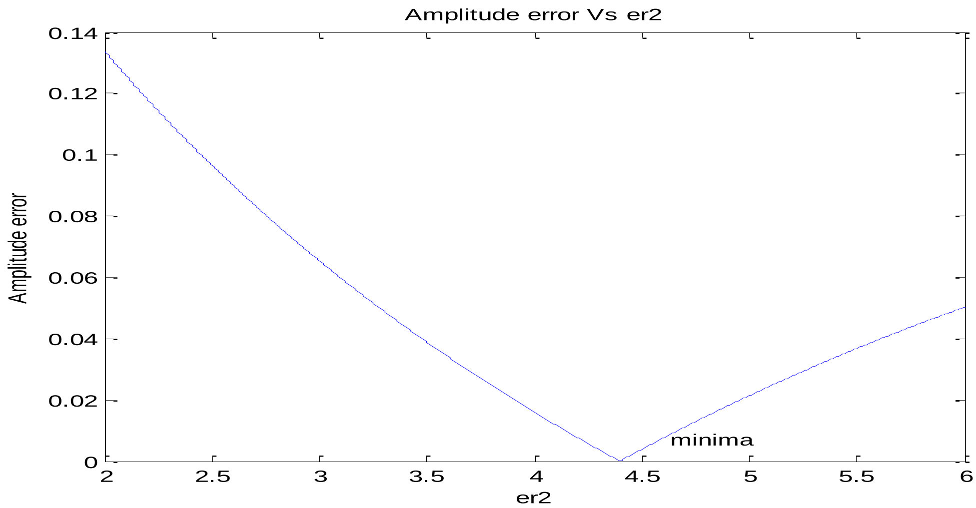

Figure 3.9 Variation of Amplitude error with respect to third layer permittivity

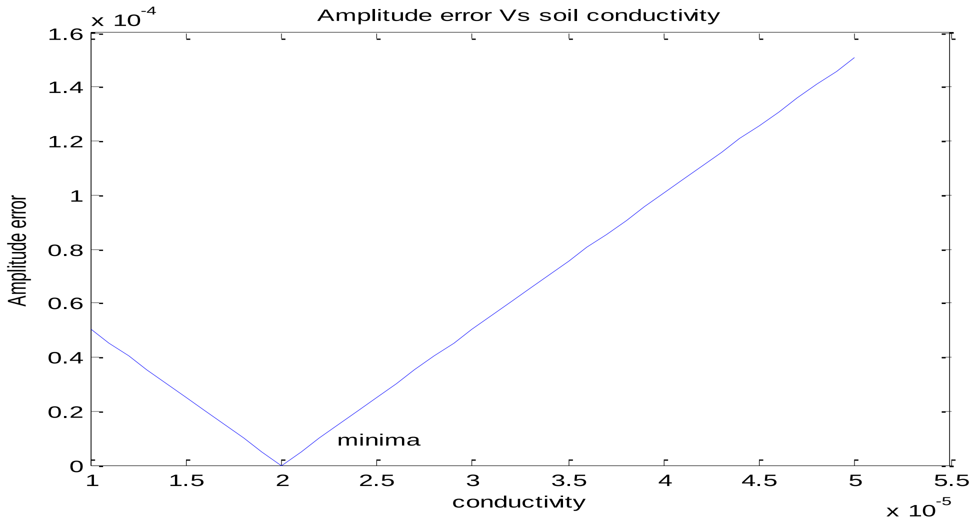

Figure 3.10 Variation of Amplitude error with respect to soil layer conductivity

3.4.2 Variation of Phase error function with subsurface parameters

Objective function defined as an error function in terms of reflection coefficient phase is called phase error function and is given as:

$$O_b(\varepsilon,\sigma,T,t_0) = \sqrt{\frac{1}{f_H - f_L}\left(\sum_{f=f_L}^{f_H}(\rho_{modeled\ phase} - \rho_{actual\ phase})^2\right)} \tag{3.8}$$

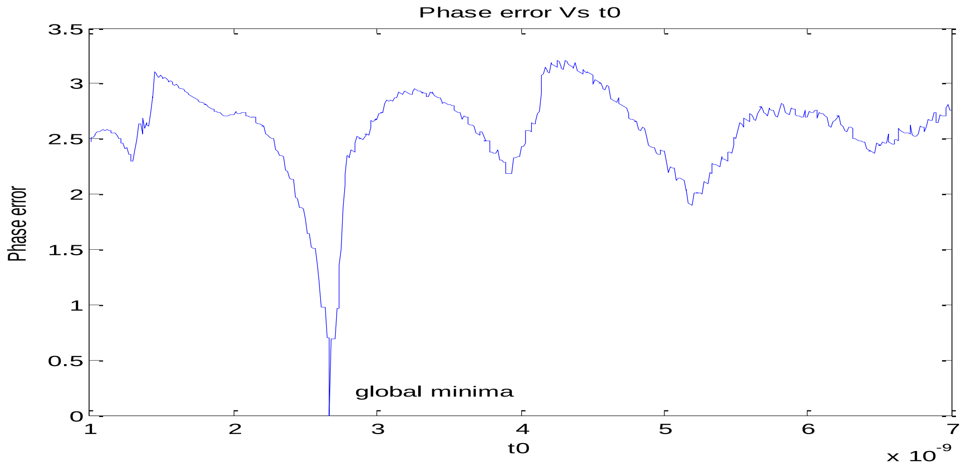

Figure 3.11 Variation of Phase error with respect to two way travel time of EM wave in air media

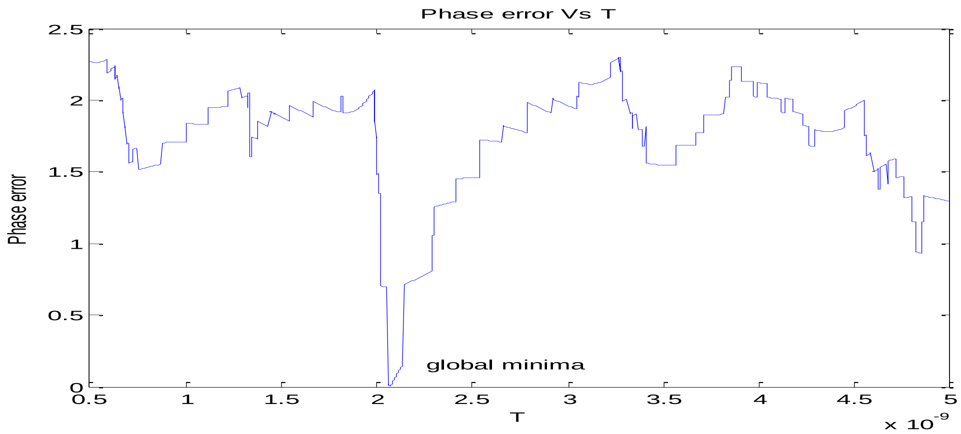

Figure 3.12 Variation of Phase error with respect to two way travel time of EM wave in soil media

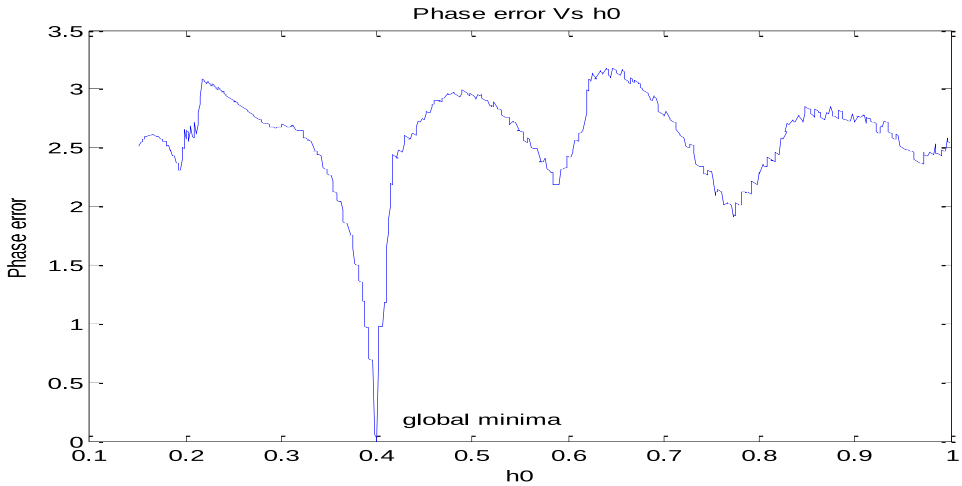

Figure 3.13 Variation of Phase error with respect to height of air layer

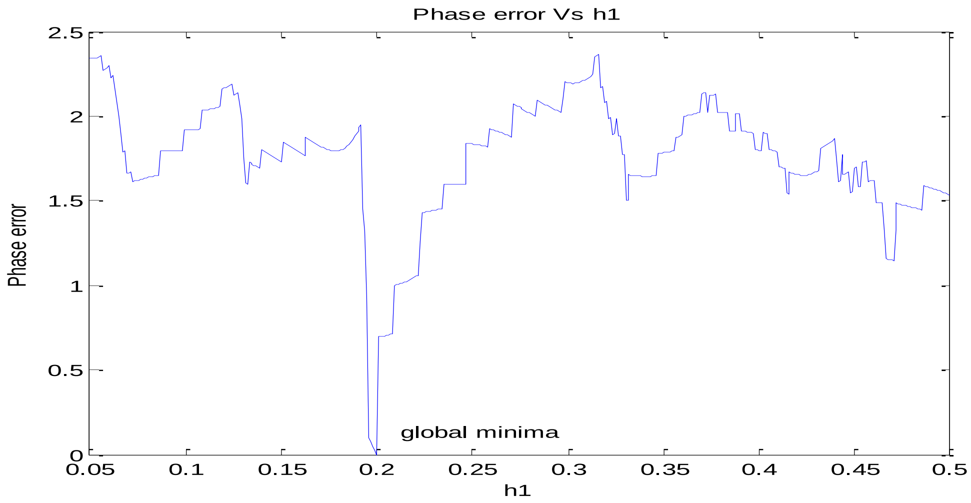

Figure 3.14 Variation of Phase error with respect to height of soil layer

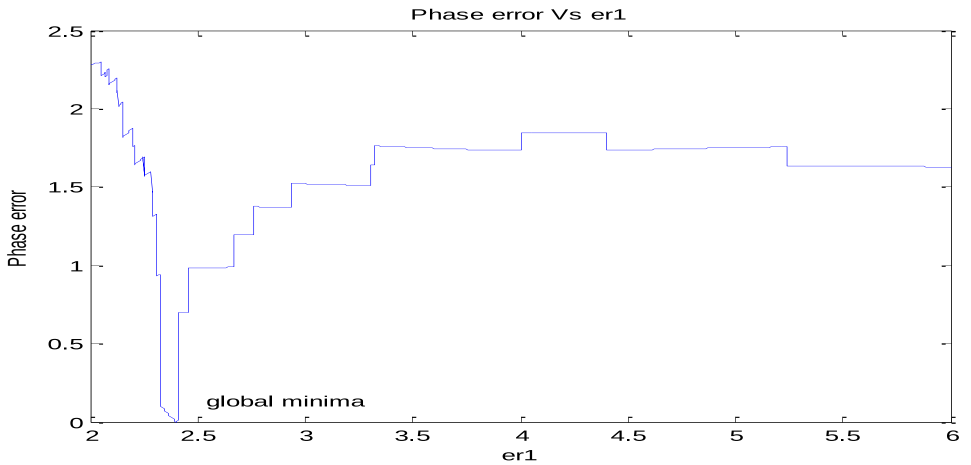

Figure 3.15 Variation of Phase error with respect to soil permittivity

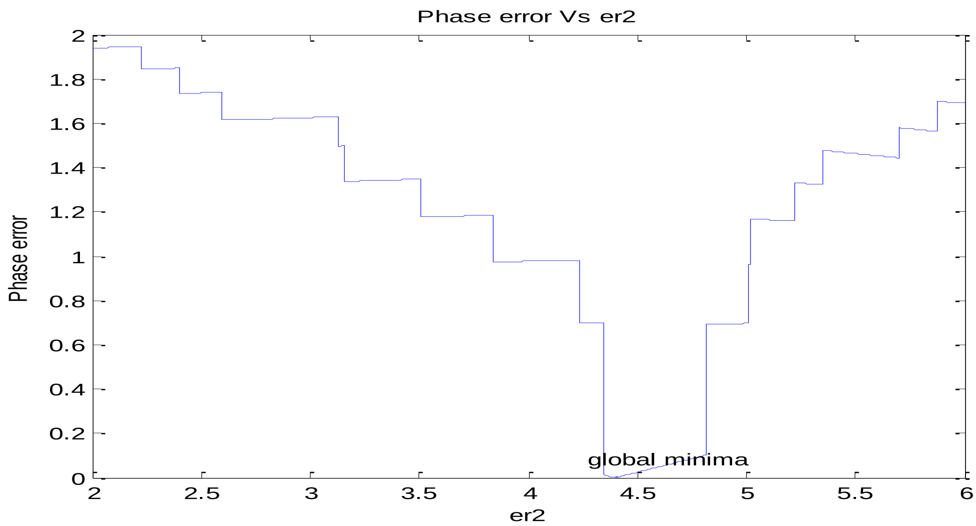

Figure 3.16 Variation of Phase error with respect to third layer permittivity

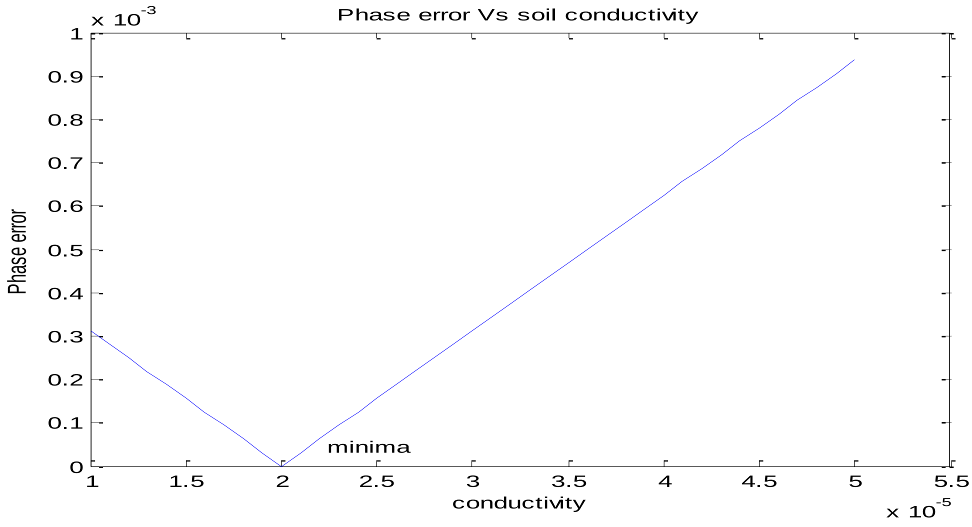

Figure 3.17 Variation of Phase error with respect to soil conductivity

Height variation (two way travel time variation) causes resonance effect for both reflection coefficient amplitude as well as phase, this account for occurrence of multiple minima in error function, where global minima represent actual subsurface parameters. Multiple minima are also observed in reflection coefficient phase error due to variation of subsurface medium electric properties i.e. relative permittivity.

Electric properties i.e. relative permittivity, conductivity variation causes local minima for reflection coefficient amplitude error. Hence subsurface electric properties can be obtained easily from reflection coefficient amplitude error by applying any local optimization techniques, provided some priory information is given about other parameters responsible for global minima.

Objective functions are formed to establish relation of GPR response with subsurface parameters and hence there can be many ways possible to define this relationship. In this thesis two different definition of objective function is presented and effectiveness of one over another is also studied in the following sections. One way to define objective function is already given by equations (3.7) and (3.8). Another form of it can be expressed mathematically as:

$$O_b(\varepsilon,\sigma,t_0,T) = \sum_{f=f_L}^{f_H} abs[Re_{\rho-actual} - Re_{\rho-modeled}] + abs[Im_{\rho-actual} - Im_{\rho-modeled}]$$

…. (3.9)

3.5 Parameters Extraction Results

As obvious from the figures representing nature of objective function with respect to surface parameters, most of the parameters are causing occurrence of global minima. Hence global optimization techniques named Pattern Search and GA-NMS algorithm is applied for parameters inversion using objective function as defined by equation (3.9). For the first case, starting point is taken at random for optimization and the result of parameters inversion is as shown below in the table:

Parameters	Range of parameters	Actual value	Initial value	Inversion results (GA-NMS)	Inversion results (Pattern Search)
ε_1	2-6	2.4	3	2.4	2
ε_2	2-7	4.4	5.5	4.3989	2.7534
σ_1(mS/m)	0.01-0.05	0.02	0.01	0.0113	0.0174
$t_0(nsec)$	1-7	2.67	3	2.67	4.8105
T(nsec)	0.5-5	2.07	1	2.07	1.9860
Time of Convergence (sec)				10.2874	10.6118

Table 2: Results of Inversion starting with random parameters value

From the table, it is clear that hybrid approach of GA-NMS algorithm is converging with accuracy for random initialization of starting point. But Pattern search is having poor accuracy. Therefore a step by step inversion methodology is adopted to obtain parameters for both the approaches. Initial value is given by the staged inversion method as discussed below:

It is done in step by step manner. The amplitude and phase spectrum are drawn corresponding to modeled and actual values of parameters taken. T is estimated first from amplitude spectrum by calculating frequency notch between its minimum points [10]. As shown in figure (3.5) and (3.12), T is responsible for occurrence of global minima. Hence estimating T from frequency notch of amplitude spectrum actually brings it in global basin. Similar is the case for t0 which can be estimated from frequency rate of change of phase spectrum as discussed in next section. Hence after eliminating global effect of T and t0, other parameters can be extracted with improved accuracy and efficiency. After completion of extraction, modeled spectrum matches actual spectrum if the parameters values are inverted accurately. The step-by-step process of parameter extraction is discussed as below:

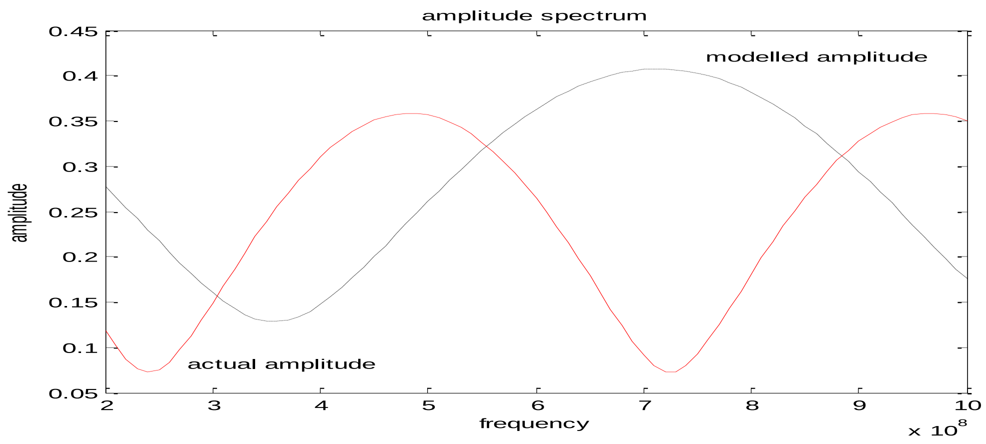

Figure 3.18 Actual Vs. Modeled Amplitude Spectrum

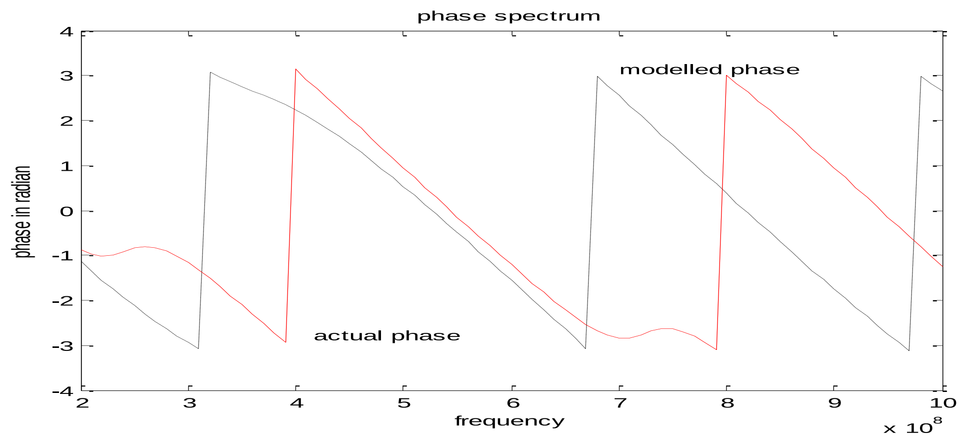

Figure 3.19 Actual Vs. Modeled Phase Spectrum

Estimation of T:

Square of amplitude spectrum of reflection coefficient (Eq. (3.5)) is given as:

$$|\rho(f)|^2 = 2({r_e}^2 + {r_o}^2) - 2\cos(2\pi fT)\,({r_o}^2 - {r_e}^2) \tag{3.10}$$

Taking derivative of equation (3.10) with respect to frequency (f)

$$\frac{d|\rho(f)|^2}{df} = 4\pi T({r_o}^2 - {r_e}^2)\sin(2\pi fT) \tag{3.11}$$

Derivative of function is zero at the point of inflection. Hence

$$\frac{d|\rho(f)|^2}{df} = 0, \tag{3.12}$$

Solution to the equation (3.12) can be given as:

$$f = (n-1)/2T, n = 1, 2, 3 \dots\dots. \tag{3.13}$$

Where f represents the frequency point where reflection coefficient amplitude is either maximum or minimum.

From figure (3.2), it is clear that amplitude spectrum is periodic in nature. Hence a frequency point corresponding to maximum value of reflection coefficient amplitude lies between two consecutive minimum points. Using this concept the frequency distance between two consecutive minimum amplitude points can be given as:

$$\Delta f = \frac{1}{T} \tag{3.14}$$

Hence T can be estimated by calculating frequency notch between two minimum points from the actual amplitude spectrum.

T estimated = 2.0833 nsec.

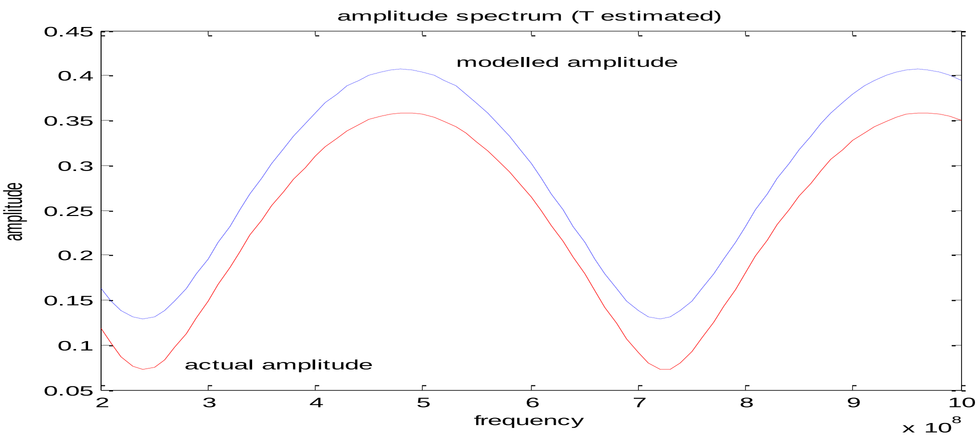

Figure 3.20 Actual Vs. Modeled Amplitude Spectrum after estimation of T

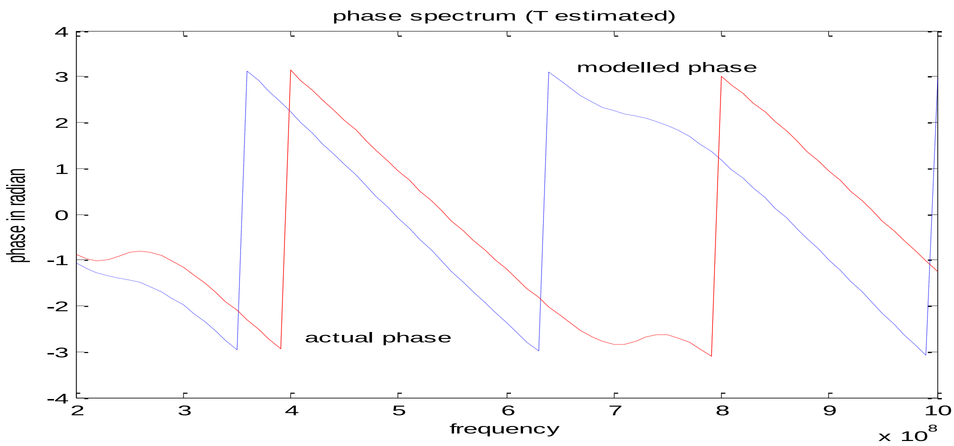

Figure 3.21 Actual Vs Modeled Phase Spectrum after estimation of T

Estimation of t0:

Phase spectrum of reflection Coefficient can be divided in two parts:

$$\angle\rho(f) = \theta_1 + \theta_2, (\theta_1, \theta_2 \in [-\pi/2, \pi/2]) \tag{3.15}$$

Where, $$\theta_1 = \arctan[\tan(\pi f T)\, r_o/r_e] \tag{3.16}$$

$$\theta_2 = \arctan[-\tan(2\pi f t_0 + \pi f T)] \tag{3.17}$$

It is clear from the above expressions that rate of change of phase with respect to frequency is a function of T, r_o/r_e and t_0. Since T is already estimated and r_o/r_e has not significant effect on frequency distance of inflection point of phase spectrum as shown below:

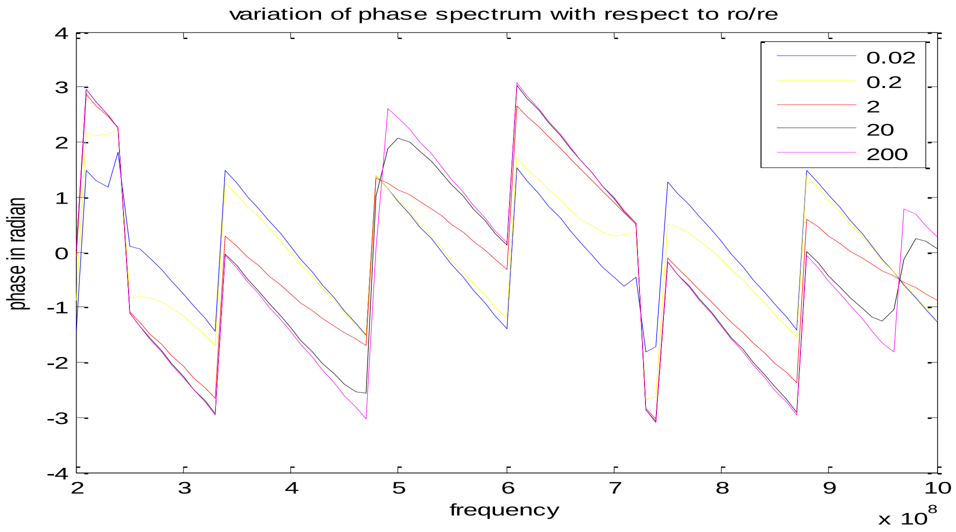

Figure 3.22 Variation of phase spectrum with respect to r_o/r_e

The only variable affecting rate of change of phase spectrum is t_0. Hence t_0 can be estimated from the rate of change of actual phase with respect to frequency.

From equation (3.15),

Rate of change of $\angle\rho(f)$ = rate of change of θ_1 + rate of change of θ_2 (3.18)

From equation (3.17),

Rate of change of $\theta_2 = -\pi T - 2\pi t_0$ (3.19)

Hence using equation (3.18) and (3.19)

$$t_0 = \frac{\frac{d\theta_1}{df} - \pi T - \frac{d\angle\rho(f)}{df}}{2\pi} \tag{3.20}$$

Hence having known the estimated value of T, t0 can be easily estimated from actual phase spectrum using equation (3.20).

t0 estimated = 2.6718 nsec.

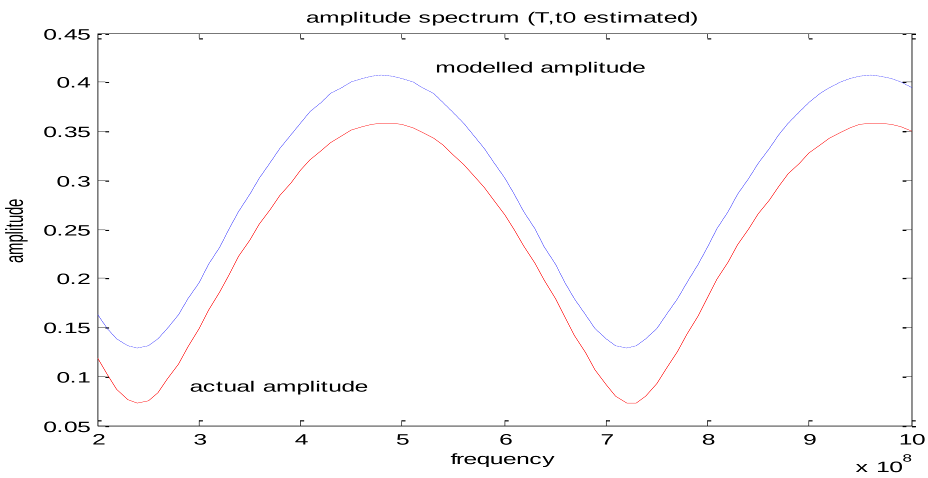

Figure 3.23 Actual Vs Modeled Amplitude Spectrum after estimation of T, t0

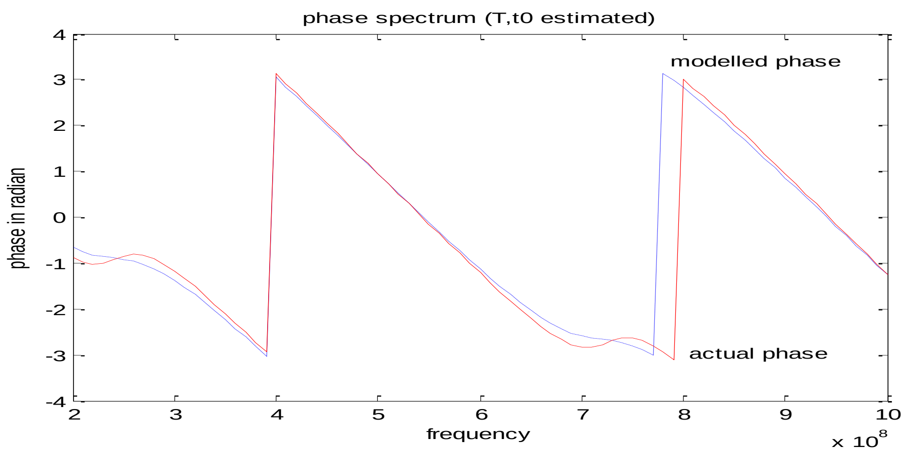

Figure 3.24 Actual Vs Modeled Phase Spectrum after estimation of T, t0

Parameters	Range of parameters	Actual value	Initial value	Inversion results (GA-NMS)	Inversion results (Pattern Search)
ε_1	2-6	2.4	3	2.4	2.4
ε_2	2-7	4.4	5.5	4.3988	4.4039
σ_1(mS/m)	0.01-0.05	0.02	0.01	0.0114	0.0494
$t_0(nsec)$	1-7	2.67	2.6718	2.67	2.67
T(nsec)	0.5-5	2.07	2.0833	2.07	2.07
Time of convergence(sec)				6.1130	25.2543

Table 3: Parameter results of inversion starting with initial values given by step by step inversion method

It is clear by comparing Table 2 and Table 3 that initial values of parameters have very negligible effect on the optimized parameters values using GA-NMS algorithm but it highly affect the accuracy of Pattern Search optimization results. However initial values of parameters have significant effect on time taken by the hybrid approach of optimization (GA-NMS) and it is clear from the Table 3 as well as figures shown below that step by step inversion has improved the efficiency of inversion process by GA-NMS technique in terms of convergence time:

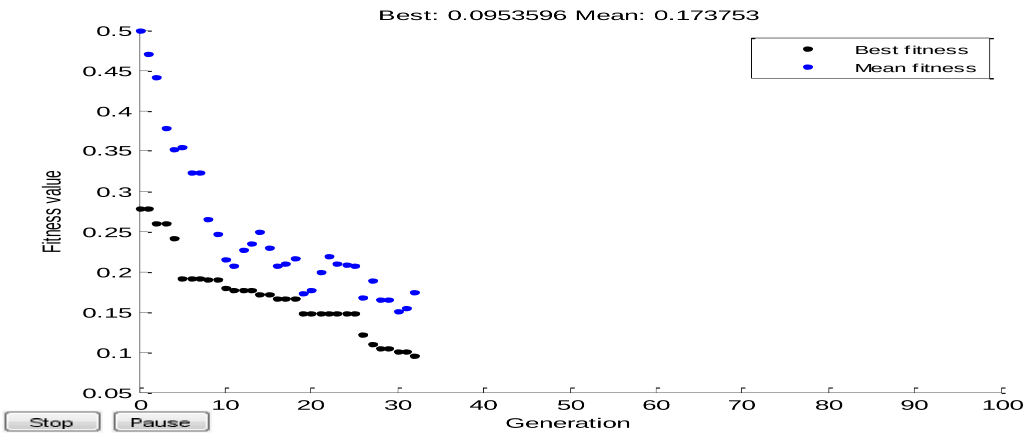

Figure 3.25 No of generation for GA with random initial parameters

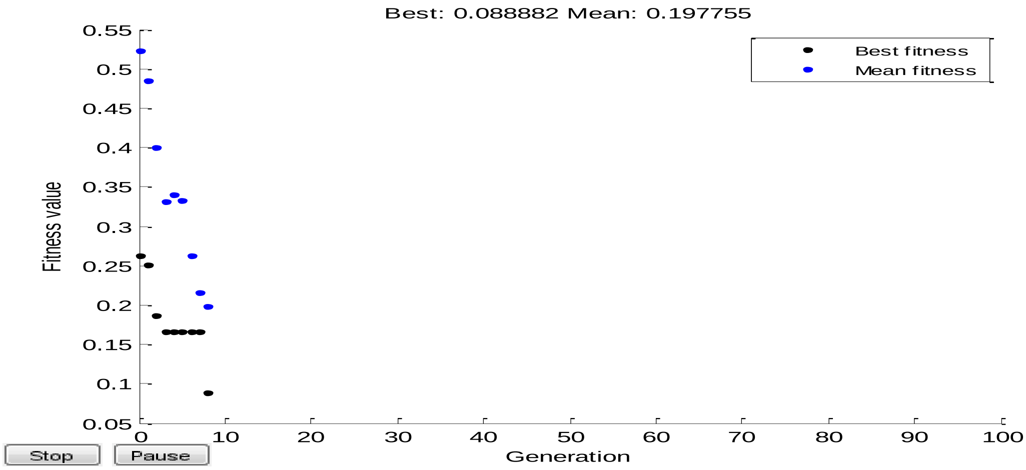

Figure 3.26 No of generation for GA with initial parameters obtained by step by step inversion technique

Effectiveness of two different approaches taken to define objective function for the inversion process is discussed and shown below using Pattern Search optimizing tool. For the objective function defined by equation (3.9) results are already obtained and displayed in Table 3. Here the discussion is presented to extract subsurface parameters using the objective function defined by equations (3.7) and (3.8).

Amplitude value is negligibly affected by t0 estimation whereas it affects phase value by considerable amount as clear by comparing figure 3.20 and figure 3.23. Hence t0 value is not affected by inversion of amplitude error function using pattern search optimization, but other parameters can be inverted with accuracy.

The parameters value obtained by inverting amplitude error function is used as initial point in inversion of t0 from phase error function and the result of inversion is as shown below in the table:

Parameters values				
Parameters types	**Actual value**	**Initial value**	**Inversion result "1"**	**Inversion result "2"**
ε_1	**2.4**	3	2.4	2.4
ε_2	**4.4**	5.5	4.4039	4.4039
σ_1 (mS/m)	**0.02**	0.01	0.0494	0.0494
t_0 (nsec)	**2.67**	2.6718	2.67	2.67
T (nsec)	**2.07**	2.0833	2.07	2.07
Time of convergence (sec)			34.5034	25.2543

Table 4 Parameters values obtained by Pattern Search optimization tool for two different objective functions

Where Inversion result "1" is parameters values when objective function is defined separately as amplitude and phase error functions by equations (3.7) and (3.8) and Inversion result "2" is parameters values when objective function is defined by equation (3.9) evaluating amplitude and phase error together by considering real and imaginary part of complex reflection coefficient. Though results obtained by using two different form of objective functions is same however they differ in efficiency in terms of number of iterations or in terms of time of convergence as shown in the Table 4 and by the figures below:

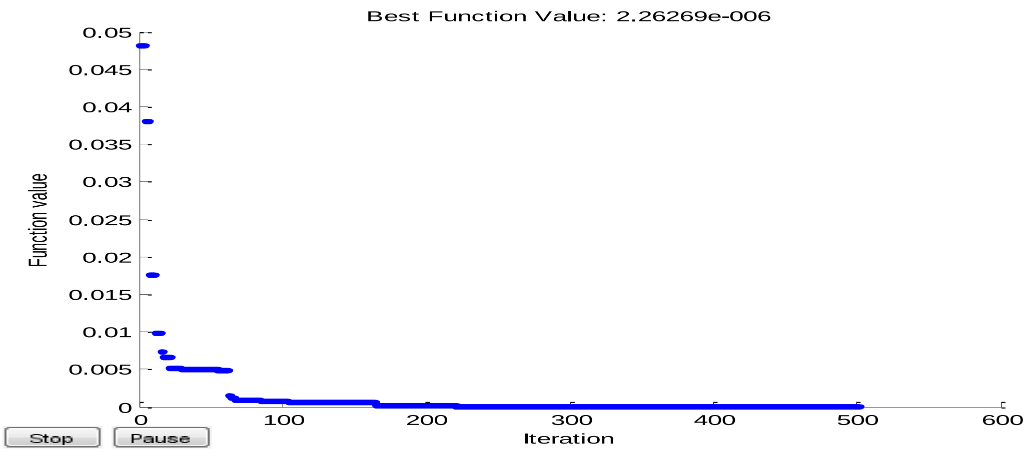

Figure 3.27 No of iterations when objective function is defined seperately in terms of amplitude and phase error function by eq (3.7) and eq (3.8)

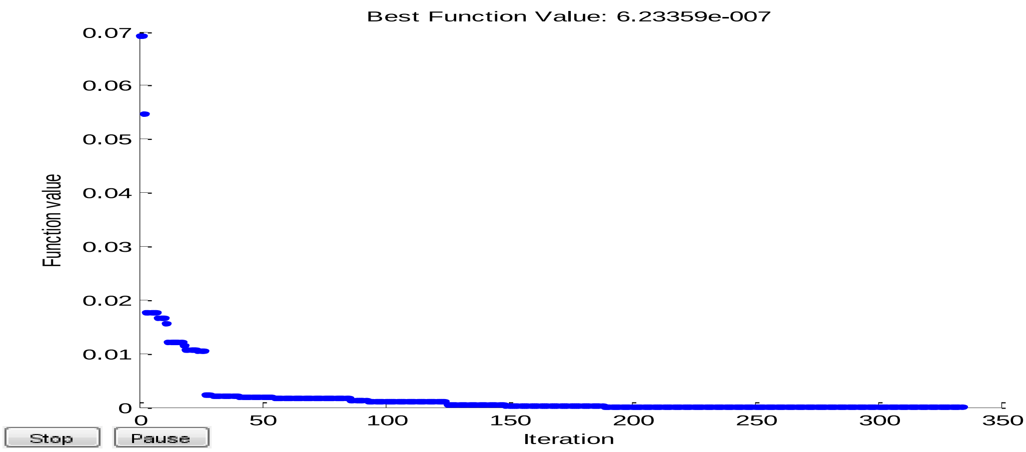

Figure 3.28 No of iterations when objective function is defined considering real and imaginary part of reflection coefficient and given by eq (3.9)

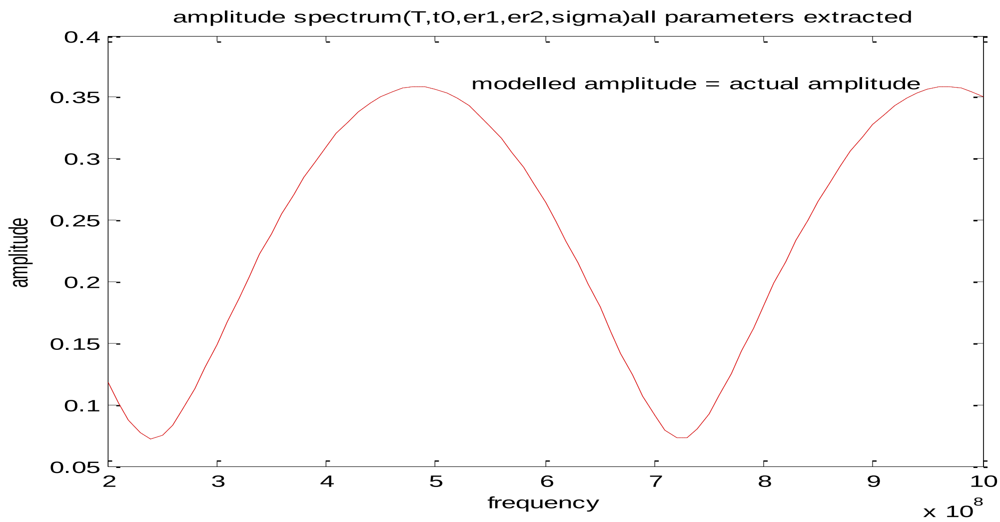

Figure 3.29 Actual vs. Modeled Amplitude spectrum after inversion of all parameters

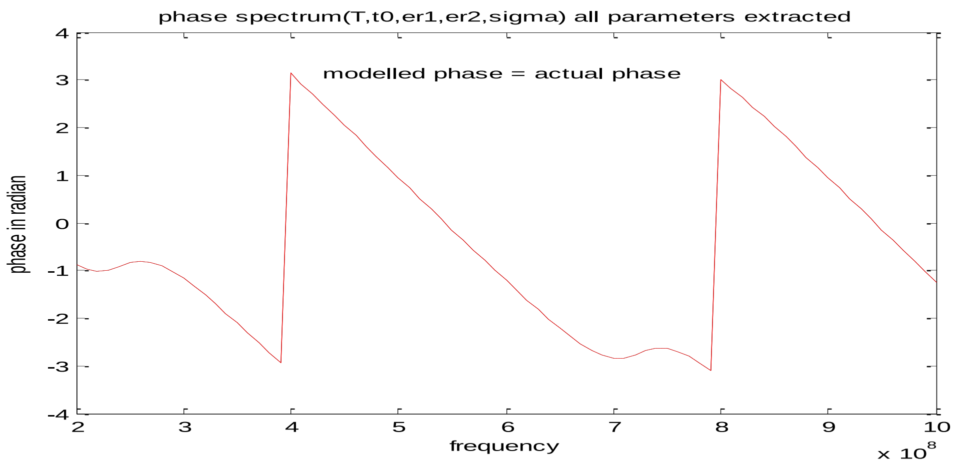

Figure 3.30 Actual Vs Modeled Phase Spectrum after inversion of all parameters

3.6 Conclusion

It is clear from amplitude and phase error curve that T and t0 are responsible for multiple minima. Amplitude spectrum is not much affected by two way travel time in air but it highly affects phase value. Hence error in t0 estimation results in high offset of phase spectrum as compared to that in amplitude spectrum. Estimating T and t0 from their impact on the reflection coefficient spectrum gives better initial values for the parameters. This process helps to start the optimization process with parameter vector locating in the global basin. Hence after estimation of T and t0 by step by step process, the efficiency and the accuracy of the optimization results are improved. Since two way travel time of EM wave in soil medium and permittivity of the medium are extracted with accuracy hence the depth of the target below the ground surface can be estimated accurately.

Hence it can be concluded from the above discussion that GA combined with a local optimizing technique is very much effective to optimize the nonlinear objective function irrespective of starting values of parameters. However convergence speed is much faster if starting values of the parameters are located in the global basin. It can also be inferred that way of defining an objective function has noticeable effect on the efficiency of inversion technique incorporated.

Chapter 4
SUMMARY & CONCLUSION

4.1 Summary

GPR is getting increased research focus due to its growing applications areas. In this research work we have implemented a standard modeling method and inversion schemes to realize a SFCW GPR. There are many GPR modeling schemes published by the researchers worldwide. Each scheme has its own advantages and disadvantages. Some of them are application specific i.e. gives more accurate parameters estimation as compared to other modeling methods for a particular application. The GPR system performance directly depends on the accuracy and efficiency of the modeling scheme. In this project work a simple surface reflection coefficient modeling approach is adopted for analysis of three layered medium consisting of air, dry soil and a third medium whose nature can be defined depending upon the application. Based on the analysis of propagation of EM wave an analytical modeling scheme of GPR wave propagation is derived. A global reflection coefficient concept is defined to take care of multiple reflections due to multilayered subsurface. An objective function is formulated as difference between actual reflection coefficient and modeled reflection coefficient. Subsurface parameters have nonlinear relation with objective function depicted by the figures showing variation of objective function with respect to subsurface parameters which causes occurrence of multiple minima. Hence two different approach consisting of global optimization approach (Pattern Search) and hybrid approach (GA-NMS) is adopted for model inversion. The two methods adopted for inversion are compared side by side. Hybrid approach is found converging for random initiation of start vector but pattern search is not converging accurately for sufficient number of parameters with random start vector. Hence a different step by step technique is adopted to make both the optimization scheme converging with good accuracy and improved speed. The result presented in Table 3 shows the improvement of convergence speed with the step by step approach implemented. The results of table 4 suggest that the selection of objective function plays an important role to estimate the soil parameters correctly.

4.2 Conclusion

The proposed modeling and inversion scheme has to be validated with the practical test cases. In this thesis a simple geometry of subsurface is considered for synthetic data and results are found to be accurate. However the presented model and inversion techniques need further analysis and sensitivity testing for different configurations of three layered sub-surface medium. The accuracy of modeling need to be verified for the various test cases like varying thickness of layers, varying step of changes in soil parameters between the different soil layers, roughness of soil surface etc. The convergence time of parameters inversion process is reduced by adopting step by step parameter estimation technique combined with hybrid approach of inversion in terms of seconds. However it requires more refinement in inversion process as a real time GPR application demands the convergence speed of the order of milliseconds. Two different optimizing methods named Pattern Search method and GA-MNS method are implemented and compared in this thesis work. Both are found to be accurate but hybrid approach is found to be more efficient as compared to the other global techniques in terms of time required for convergence of the optimizing algorithms. The GPR modeling and inversion scheme proposed in this thesis can be employed for the GPR applications like metallic or nonmetallic landmine detection, water content estimation etc. after test validating the model in laboratory environment.

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